



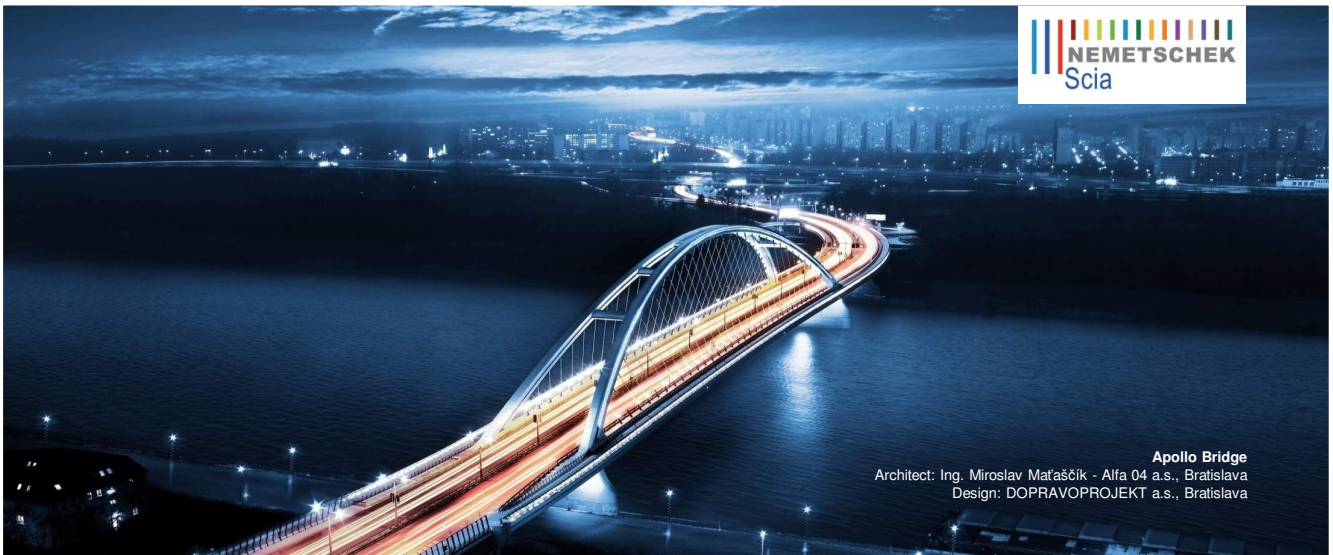
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Eurocode Training

EN 1992-1-1: Reinforced Concrete

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Eurocode Training

EN 1992-1-1: Reinforced Concrete

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Introduction

Subject of this workshop

Subject of this workshop = the European Standard EN 1992

Eurocode 2: Design of concrete structures

Part 1-1: General rules and rules for buildings

which has been prepared by Technical Committee CEN/TC250 «Structural Eurocodes»

Introduction

Code settings in Scia Engineer



The screenshot displays three overlapping windows in the Scia Engineer software interface:

- Project data**: A window with tabs for 'Basic data', 'Functionality', 'Loads', and 'Protection'. The 'Data' sub-tab is active, showing fields for Name, Part, Description, Author, and Date (09.04.2010). A 'Material' section on the right lists options like Concrete, Material (C20/25), Reinforcement mat. (B 500A), Steel, Timber, Other, and Aluminium.
- Annex code**: A window showing a grid of national flags and codes. The 'United Kingdom' is selected at the bottom. A 'Code' sub-window is overlaid on this, with 'National Code' and 'National annex' both set to 'EC-EN'.
- Manager for National annexes**: A window showing a tree view of design codes. The 'EC-EN' code is selected, and its sub-items are expanded, including 'EN 1990: Basic of structural design', 'EN 1991: Actions of structures', 'EN 1992: Design of concrete structures', 'EN 1993: Design of steel structures', 'EN 1994: Design of composite steel and concrete structures', 'EN 1997: Geotechnical design', and 'EN 1999: Design of aluminium structures'.

Section 1 General



Section 1 General

Scope of Eurocode 2



- Design of buildings and civil engineering works in plain, reinforced and prestressed concrete
- Requirements for resistance, serviceability, durability and fire resistance of concrete structures
- Eurocode 2 is subdivided into the following parts:
 - Part 1-1: General rules and rules for buildings
 - Part 1-2: Structural fire design
 - Part 2: Reinforced and prestressed concrete bridges
 - Part 3: Liquid retaining and containing structure

- **Focus in this workshop: Reinforced concrete – Part 1-1**

Section 1 General

Scope of Part 1-1 of Eurocode 2



- General basis for design of structures in plain, reinforced and prestressed concrete made with normal and light weight aggregates together with specific rules for buildings
 - Section 1: General
 - Section 2: Basis of design
 - Section 3: Materials
 - Section 4: Durability and cover to reinforcement
 - Section 5: Structural analysis
 - Section 6: Ultimate limit states
 - Section 7: Serviceability limit states
 - Section 8: Detailing of reinforcement and prestressing tendons - General
 - Section 9: Detailing of members and particular rules

 - Section 10: Additional rules for precast concrete elements and structures
 - Section 11: Lightweight aggregate concrete structures
 - Section 12: Plain and lightly reinforced concrete structures

- **Focus in this workshop: Sections 1 to 9**

Section 2 Basis of design

Section 2 Basis of design

Basic variables

Material and product properties

- general rules → see EN 1990 Section 4
- specific provisions for concrete & reinforcement → see EN 1992 Section 3

Shrinkage and creep

- time dependent
- effects have to be taken into account in the SLS
- in the ULS: only if significant (for example 2nd order)
- quasi-permanent combination of loads

Application in Scia Engineer:

Creep: CDD or PNL calculation
Shrinkage: TDA calculation

Section 2 Basis of design

Verification by the partial factor method

Design values

Partial factors for shrinkage, prestress, fatigue loads

Partial factors for materials

- ULS: recommended values

Design situations	γ_c for concrete	γ_s for reinforcing steel	γ_s for prestressing steel
Persistent & Transient	1,5	1,15	1,15
Accidental	1,2	1,0	1,0

- SLS: recommended values

$$\gamma_c \text{ and } \gamma_s = 1$$

Section 2 Basis of design

Verification by the partial factor method

Design values

Partial factors in Scia Engineer

Name	EC-EN
Concrete	EC-EN
Design defaults	
General	
Concrete	
National annex	
EN_1992_1_1	
gamma_c_per - partial factor for concrete, ULS, persistent and transient de...	1,50
gamma_c_acc - partial factor for concrete, ULS, accidental design situation...	1,20
fck_max - maximum value of the characteristic cylinder strength 3.1.2(2) [M...	90,00
alpha_cc - coeff. taking account of long term effects on the compressive stre...	1,00
alpha_ct - coeff. taking account of long term effects on the tensile strength 3...	1,00

Name	EC-EN
Concrete	EC-EN
Design defaults	
General	
Concrete	
Non-prestressed reinforcement	
National annex	
EN_1992_1_1	
gamma_s_per - partial factor for ULS, persistent design situation 2.4.2.4(1) [-]	1,15
gamma_s_acc - partial factor for ULS, accidental design situation 2.4.2.4(1) [-]	1,00
eps_ud/eps_uk - ratio of design and characteristic strain limit 3.2.7(2) [-]	0,90
EN_1992_1_2	
gamma_s_fi - partial factor for ULS, fire situation 2.3(2)P [-]	1,00

Section 3 Materials

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Section 3 Materials

Concrete

Characteristic strength

Compressive strength

- denoted by concrete strength classes (e.g. C25/30)
- cylinder strength f_{ck} and cube strength $f_{ck,cube}$
- f_{ck} determined at 28 days

If required to specify $f_{ck}(t)$ at time t for a number of stages
(e.g. demoulding, transfer of prestress):

- $f_{ck}(t) = f_{cm}(t) - 8$ [MPa] for $3 < t < 28$ days
- $f_{ck}(t) = f_{ck}$ for $t \geq 28$ days

where $f_{cm}(t) = \beta_{cc}(t) f_{cm}$ with $\beta_{cc}(t)$ dependent on the cement class

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Section 3 Materials

Concrete



EN Table 3.1 Strength and deformation characteristics for concrete

Strength classes for concrete														Analytical relation / Explanation					
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90					
$f_{d,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105					
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98	$f_{cm} = f_{ck} + 8$ (MPa)				
f_{cm} (MPa)	1.6	1.9	2.2	2.6	2.9	3.2	3.5	3.8	4.1	4.2	4.4	4.6	4.8	5.0	$f_{cm} = 0.30 \cdot f_{ck}^{(0.75)} \leq C50/60$ $f_{cm} = 2.12 \cdot \ln(1 + (f_{ck}/10))$ $> C50/60$				
$f_{ak,0.05}$ (MPa)	1.1	1.3	1.5	1.8	2.0	2.2	2.5	2.7	2.9	3.0	3.1	3.2	3.4	3.5	$f_{ak,0.05} = 0.7 \cdot f_{cm}$ 5% fractile				
$f_{ak,0.95}$ (MPa)	2.0	2.5	2.9	3.3	3.8	4.2	4.6	4.9	5.3	5.5	5.7	6.0	6.3	6.6	$f_{ak,0.95} = 1.3 \cdot f_{cm}$ 95% fractile				
E_{cm} (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44	$E_{cm} = 22 \cdot (f_{cm}/10)^{1.5}$ (f_{cm} in MPa)				
ϵ_{c1} (‰)	1.8	1.9	2.0	2.1	2.2	2.25	2.3	2.4	2.45	2.5	2.6	2.7	2.8	2.8	see Figure 3.2 $\epsilon_{c1}(f_{cm}) = 0.7 \cdot f_{cm}^{0.01} < 2.8$				
ϵ_{c1} (‰)					3.5								3.2		3.0	2.8	2.8	2.8	see Figure 3.2 for $f_{ck} \geq 50$ Mpa $\epsilon_{c1}(f_{cm}) = 2.8 + 27 \cdot (98 - f_{cm}) / 100$
ϵ_{c2} (‰)					2.0								2.2		2.3	2.4	2.5	2.6	see Figure 3.3 for $f_{ck} \geq 50$ Mpa $\epsilon_{c2}(f_{cm}) = 2.0 + 0.085 \cdot (f_{ck} - 50)^{0.5}$
ϵ_{c2} (‰)					3.5								3.1		2.9	2.7	2.6	2.6	see Figure 3.3 for $f_{ck} \geq 50$ Mpa $\epsilon_{c2}(f_{cm}) = 2.6 + 35 \cdot (90 - f_{ck}) / 100$
n					2.0								1.75		1.6	1.45	1.4	1.4	for $f_{ck} \geq 50$ Mpa $n = 1.4 + 23.4 \cdot (90 - f_{ck}) / 100$
ϵ_{c3} (‰)					1.75								1.8		1.9	2.0	2.2	2.3	see Figure 3.4 for $f_{ck} \geq 50$ Mpa $\epsilon_{c3}(f_{cm}) = 1.75 + 0.55 \cdot (f_{ck} - 50) / 40$
ϵ_{c3} (‰)					3.5								3.1		2.9	2.7	2.6	2.6	see Figure 3.4 for $f_{ck} \geq 50$ Mpa $\epsilon_{c3}(f_{cm}) = 2.6 + 35 \cdot (90 - f_{ck}) / 100$

Section 3 Materials

Concrete



Material characteristics in Scia Engineer

The screenshot shows the 'Materials' window in Scia Engineer. The material 'C30/37' is selected. The properties are as follows:

- G modulus [MPa]: 1.3667e+04
- Log decrement: 0.2
- Colour: (empty)
- Specific heat [J/gK]: 6.0000e+01
- Temperature dependency of specific heat: None
- Thermal conductivity [W/mK]: 4.5000e+01
- Temperature dependency of thermal conductivity: None
- Order in code: 5
- EN 1992-1-1**
 - Characteristic compressive cylinder strength $f_{ck}(28)$ [MPa]: 30.00
 - Calculated depended values:
 - Mean compressive strength $f_{cm}(28)$ [MPa]: 38.00
 - $f_{cm}(28) - f_{ck}(28)$ [MPa]: 8.00
 - Mean tensile strength $f_{ctm}(28)$ [MPa]: 2.90
 - $f_{ctk} 0.05(28)$ [MPa]: 2.00
 - $f_{ctk} 0.95(28)$ [MPa]: 3.80
 - Design compressive strength - persistent ($f_{cd} = f_{ck} / \gamma_{c,p}$) [MPa]: 20.00
 - Design compressive strength - accidental ($f_{cd} = f_{ck} / \gamma_{c,a}$) [MPa]: 25.00
 - Strain at reaching maximum strength ϵ_{c2} [1e-4]: 20.0
 - Ultimate strain ϵ_{cu2} [1e-4]: 35.0
 - Strain at reaching maximum strength ϵ_{c3} [1e-4]: 17.5
 - Ultimate strain ϵ_{cu3} [1e-4]: 35.0
 - Stone diameter (dg) [mm]: 32
 - Cement class: N (normal hardening - CEM 32.5 R, CEM 42.5 N)
 - Type of aggregate: Quartzite
- Measured values
 - Measured values of mean compressive strength (influence of ageing):
- Stress-strain diagram
 - Type of diagram: Bi-linear stress-strain diagram
 - Picture of Stress-strain diagram: (empty)

Design strength

Design compressive strength

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_C \quad (3.15)$$

Design tensile strength

$$f_{ctd} = \alpha_{ct} f_{ctk,0,05} / \gamma_C \quad (3.16)$$

α_{cc} (resp. α_{ct}) is a coefficient taking account of **long term effects** on the compressive strength (resp. tensile strength) and of unfavourable effects resulting from the way the load is applied.

$$\alpha_{cc} = 1,0 \text{ and } \alpha_{ct} = 1,0 \quad (\text{recommended values})$$

Elastic deformation

- dependent on composition of the concrete (especially the aggregates)
- approximate values for modulus of elasticity E_{cm} , secant value between $\sigma_c = 0$ and $0,4f_{cm}$: see EN Table 3.1 (for quartzite aggregates)
 - Reduction for limestone aggregates (10%) – sandstone aggregates (30%)
 - Augmentation for basalt aggregates (20%)

- variation of the modulus of elasticity with time:

$$E_{cm}(t) = (f_{cm}(t) / f_{cm})^{0,3} E_{cm} \quad (3.5)$$

- Poisson's ratio: 0,2 for uncracked concrete and 0 for cracked concrete

Creep and shrinkage

dependent on

- ambient humidity
- dimensions of the element
- composition of the concrete
- maturity of the concrete at first loading
- duration and magnitude of the loading

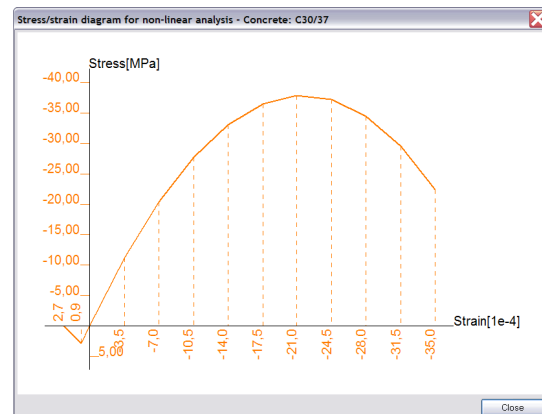
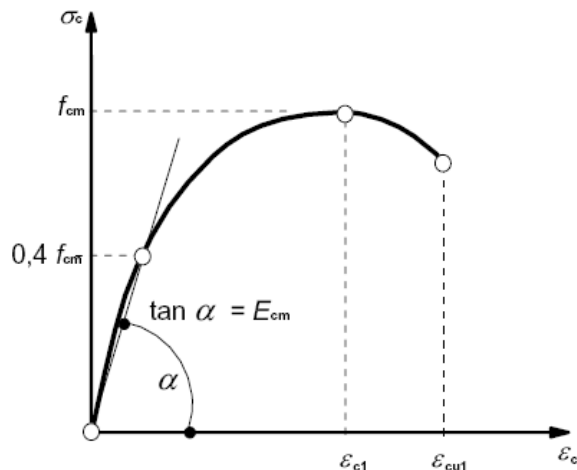
Creep coefficient in Scia Engineer

Creep	
Creep for concrete - Code dependent deflections (CDD)	
Creep coefficient [-]	2.50
Calculate creep coefficient	<input checked="" type="checkbox"/> yes
Relative humidity [%] [-]	50.00
Age at loading [day]	28
Age at concrete [day]	1825

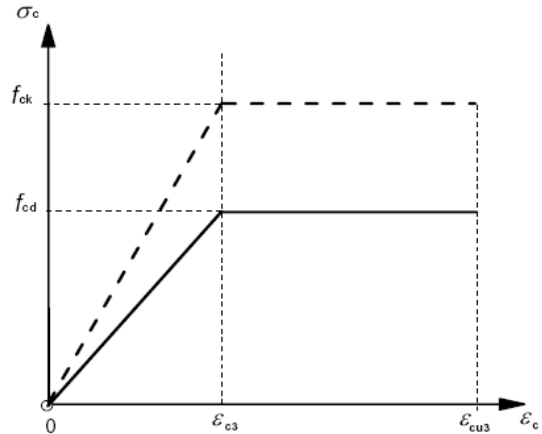
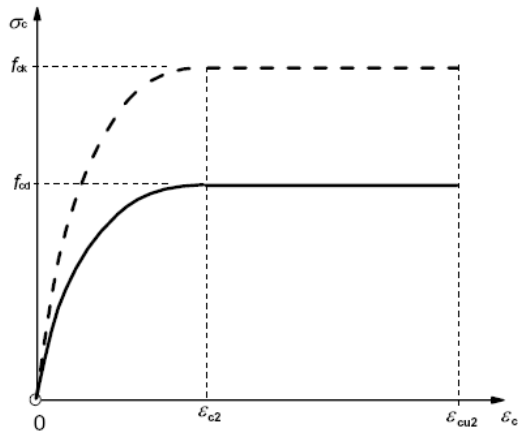
Stress-strain relation for non-linear structural analysis

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta}$$

Remark: the use of $0,4f_{cm}$ for the definition of E_{cm} is approximate!



Stress-strain relations for the design of cross-sections



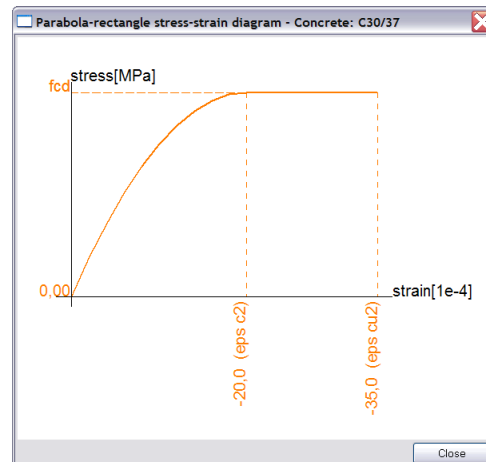
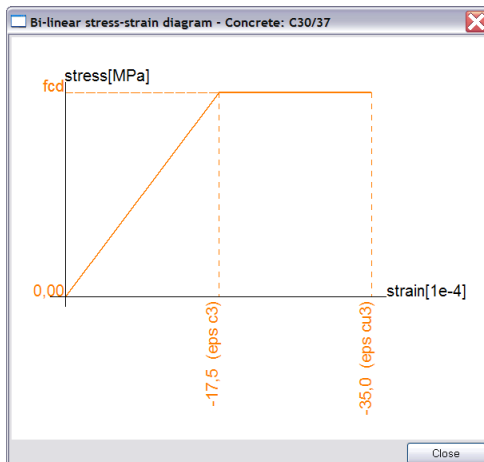
$\epsilon_{c2,3}$: strain at reaching the maximum strength

$\epsilon_{cu2,3}$: ultimate strain

(see EN Table 3.1)

Stress-strain relations for the design of cross-sections in Scia Engineer

☐ Stress-strain diagram	
Type of diagram	Bi-linear stress-strain diagram
Picture of Stress-strain diagram	Bi-linear stress-strain diagram Parabola-rectangle stress-strain diagram



Properties

The behaviour of reinforcing steel is specified by the following properties:

- yield strength (f_{yk} or $f_{0,2k}$)
- maximum actual yield strength ($f_{y,max}$)
- tensile strength (f_t)
- ductility (ϵ_{uk} and f_t/f_{yk})
- bendability
- bond characteristics (f_R : see Annex C)
- section sizes and tolerances
- fatigue strength
- weldability
- shear and weld strength for welded fabric and lattice girders

Material characteristics in Scia Engineer

Name	B 500A
Code independent	
Material type	Reinforcement steel
Thermal expansion [m/mK]	0.00
Unit mass [kg/m ³]	7850.00
E modulus [MPa]	2.0000e+05
Poisson coeff	0.2
Independent G modulus	<input type="checkbox"/>
G modulus [MPa]	8.3333e+04
Log decrement	0.2
Colour	
Specific heat [J/gK]	6.0000e-01
Thermal conductivity [W/mK]	4.5000e+01
Bar surface	Ribbed
Order in code	2
EN 1992-1-1	
Characteristic yield strength f_{yk} [MPa]	500.0
Calculated depended values	<input type="checkbox"/>
Charakteristic maximum tensile strength f_{tk} [MPa]	525.0
Coefficient $k = f_{tk} / f_{yk}$ [-]	1.05
Design yield strength - persistent (fpd = $f_{yk} / \gamma_{s,p}$) [MPa]	434.8
Design yield strength - accidental (fpd = $f_{yk} / \gamma_{s,a}$) [MPa]	500.0
Maximum elongation $\epsilon_{ps,uk}$ [1e-4]	250.0
Class	A
Reinforcement type	Bars
Fabrication	Hot rolled
Stress-strain diagram	
Type of diagram	Bi-linear with an inclined top branch
Picture of Stress-strain diagram	...

Section 3 Materials

Reinforcing Steel

- Specified yield strength range: $f_{yk} = 400$ to 600 MPa
- Specific properties for classes A – B – C: see Annex C

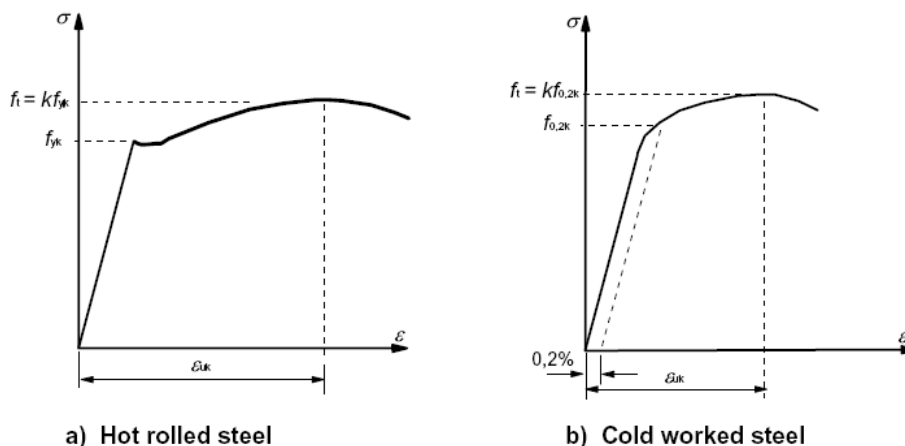
Product form	Bars and de-coiled rods			Wire Fabrics			Requirement or quantile value (%)
	A	B	C	A	B	C	
Class							-
Characteristic yield strength f_{yk} or $f_{0,2k}$ (MPa)	400 to 600						5,0
Minimum value of $k = (f_t/f_y)_k$	$\geq 1,05$	$\geq 1,08$	$\geq 1,15$ $< 1,35$	$\geq 1,05$	$\geq 1,08$	$\geq 1,15$ $< 1,35$	10,0
Characteristic strain at maximum force, ϵ_{uk} (%)	$\geq 2,5$	$\geq 5,0$	$\geq 7,5$	$\geq 2,5$	$\geq 5,0$	$\geq 7,5$	10,0
Bendability	Bend/Rebend test			-			
Shear strength	-			$0,3 A f_{yk}$ (A is area of wire)			Minimum
Maximum deviation from nominal mass (individual bar or wire) (%)	Nominal bar size (mm) ≤ 8 > 8			$\pm 6,0$ $\pm 4,5$			5,0

Class A is used by default; B and C have more rotation capacity.

Section 3 Materials

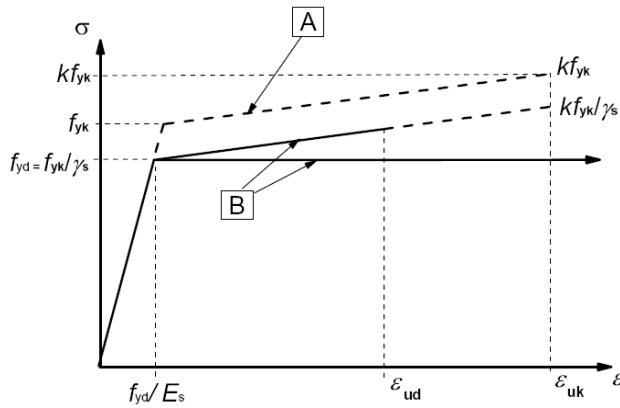
Reinforcing Steel

Stress-strain diagrams of typical reinforcing steel



- yield strength f_{yk} (or the 0,2% proof stress, $f_{0,2k}$)
- tensile strength f_{tk}
- adequate ductility is necessary, defined by the $(f_t/f_y)_k$ and ϵ_{uk}

Stress-strain diagrams for design



$k = (f_t/f_y)_k$

A Idealised

B Design

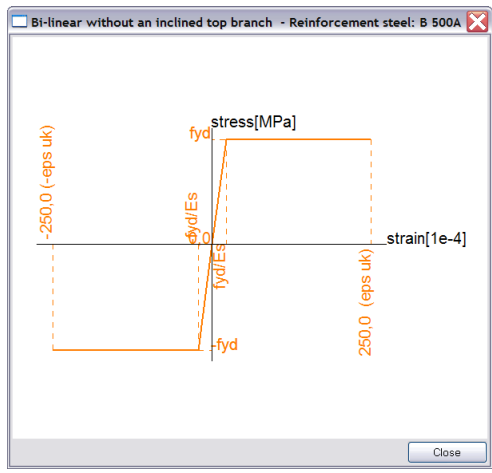
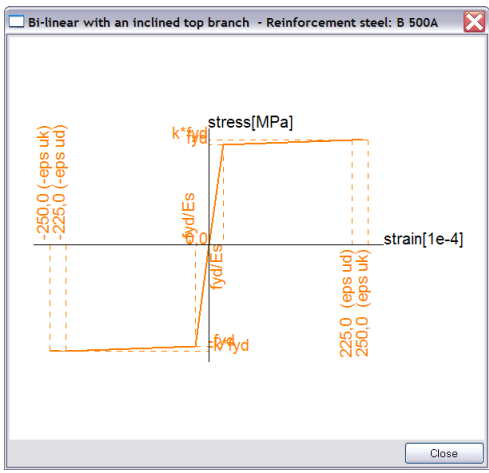
Recommended value:
 $\epsilon_{ud} = 0,9 \epsilon_{uk}$

Assumptions for design:

- (a) inclined top branch with a strain limit of ϵ_{ud} and a maximum stress of kf_{yk}/γ_s at ϵ_{uk}
- (b) horizontal top branch without the check of strain limit

Stress-strain diagrams for design in Scia Engineer

Stress-strain diagram	
Type of diagram	Bi-linear with an inclined top branch
Picture of Stress-strain diagram	Bi-linear with an inclined top branch
	Bi-linear without an inclined top branch



Section 4 Durability and cover to reinforcement

Section 4 Durability and cover to reinforcement Environmental Conditions

Class designation	Description of the environment	Informative examples where exposure classes may occur
1 No risk of corrosion or attack		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
2 Corrosion induced by carbonation		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
3 Corrosion induced by chlorides		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to chlorides Pavements Car park slabs
4 Corrosion induced by chlorides from sea water		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the sea
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures

EN Table 4.1 Exposure classes related to environmental conditions

5. Freeze/Thaw Attack		
XF1	Moderate water saturation, without de-icing agent	Vertical concrete surfaces exposed to rain and freezing
XF2	Moderate water saturation, with de-icing agent	Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents Horizontal concrete surfaces exposed to rain and freezing
XF3	High water saturation, without de-icing agents	Horizontal concrete surfaces exposed to rain and freezing
XF4	High water saturation with de-icing agents or sea water	Road and bridge decks exposed to de-icing agents Concrete surfaces exposed to direct spray containing de-icing agents and freezing Splash zone of marine structures exposed to freezing
6. Chemical attack		
XA1	Slightly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA2	Moderately aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA3	Highly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water

Concrete cover

Nominal cover

$$c_{\text{nom}} = c_{\text{min}} + \Delta c_{\text{dev}} \quad (4.1)$$

c_{min} , minimum cover, in order to ensure:

- the safe transmission of bond forces
- the protection of the steel against corrosion (durability)
- an adequate fire resistance (see EN 1992-1-2)

Δc_{dev} , allowance in design for deviation

Concrete cover

Minimum cover, c_{min}

$$c_{\text{min}} = \max \{ c_{\text{min,b}} ; c_{\text{min,dur}} + \Delta c_{\text{dur,g}} - \Delta c_{\text{dur,st}} - \Delta c_{\text{dur,add}} ; 10 \text{ mm} \} \quad (4.2)$$

where:

- $c_{\text{min,b}}$ minimum cover due to bond requirement
- $c_{\text{min,dur}}$ minimum cover due to environmental conditions
- $\Delta c_{\text{dur,g}}$ additive safety element
- $\Delta c_{\text{dur,st}}$ reduction of minimum cover for use of stainless steel
- $\Delta c_{\text{dur,add}}$ reduction of minimum cover for use of additional protection

Allowance in design for deviation, Δc_{dev}

$$\Delta c_{\text{dev}} = 10 \text{ mm} \quad (\text{recommended value})$$

Concrete cover in Scia Engineer

Exposure class	XC3
Abrasion class	X0
Type of concrete	XC1
Special quality control	XC2
Columns	XC4
Beams	XD1
2D structures and be...	XD2
Punching	XD3
Default sway type (f...	XS1
General	XS2
	XS3

Abrasion class	None
Type of concrete	None
Special quality control	XM1
Columns	XM2
Beams	XM3

Section 5 Structural Analysis

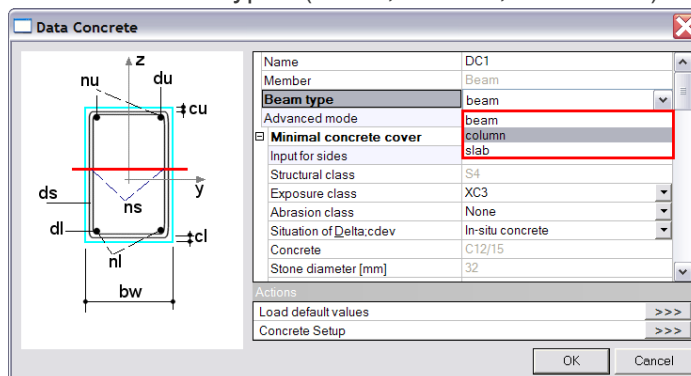
Structural models for overall analysis

The elements of a structure are classified, by consideration of their nature and function, as **beams, columns, slabs, walls, plates, arches, shells** etc. Rules are provided for the analysis of the commoner of these elements and of structures consisting of combinations of these elements.

See EN § 5.3.1 for the descriptions

Assignment of structural models in Scia Engineer

- For **1D members**: 3 types (beam, column, beam slab) – choice made by user

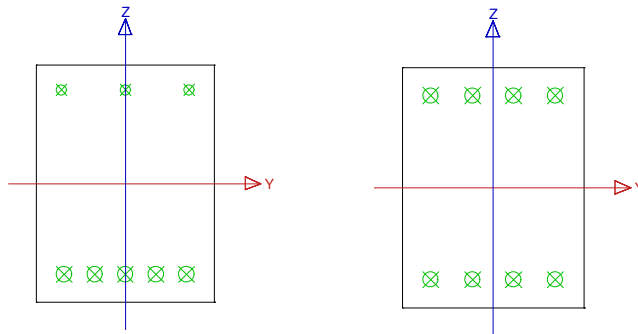


! Different calculation methods !

- For **2D members**: 3 types (plate, wall, shell) – detected by NEDIM solver, based on present internal forces

Assignment of structural models in Scia Engineer

Beam calculation ↔ Column calculation



Difference in reinforcement area per direction

Internal forces taken into account:

- Beam calculation: N, M_y, V_z
- Column calculation: N, M_y, M_z, V_z

Geometric data

Continuous slabs and beams may generally be analysed on the assumption that the supports provide no rotational restraint.

- **Monolithic connection (Fixed support)**

Critical design moment at the support = Moment at the face of the support

- **Hinged support**

Design support moment may be reduced by an amount ΔM_{Ed} :

$$\Delta M_{Ed} = F_{Ed,sup} t / 8 \quad (5.9)$$

where: $F_{Ed,sup}$ is the design support reaction
 t is the breadth of the support

Moment reduction above support in Scia Engineer

- Durability and concrete cover
- Calculation
 - General
 - Columns
 - Beams
- ULS
 - General
 - Interaction diagram

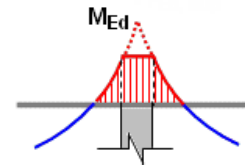
Beams	
Calculate compression reinforcement	<input checked="" type="checkbox"/> yes
Include normal force to calculation	<input checked="" type="checkbox"/> yes
Check compression of member	<input type="checkbox"/> no
$NEd < x \cdot A_c \cdot f_{cd}; x = [-]$	0.00
Moment reduction at supports	<input type="checkbox"/> no
Shear force reduction at supports	<input type="checkbox"/> no

Nodal support

Support in node

Name	Sr1
Type	Standard
Angle [deg]	
X	Rigid
Z	Rigid
Ry	Free
Default size [m]	0.200
Node	R1
Geometry	
System	GCS

Column support



Common **idealisations of the behaviour** used for analysis are:

- (a) linear elastic behaviour
- (b) linear elastic behaviour with limited redistribution
- (c) plastic behaviour, including strut and tie models
- (d) non-linear behaviour

Section 5 Structural Analysis

(a) Linear elastic analysis

- Based on the theory of elasticity
- Suitable for both ULS and SLS
- Assumptions:
 - uncracked cross-sections
 - linear stress-strain relationships
 - mean value of E
- For thermal deformation, settlement and shrinkage effects:
 - at ULS: reduced stiffness (cracking - tension stiffening + creep)
 - at SLS: gradual evolution of cracking

Linear elastic analysis in Scia Engineer

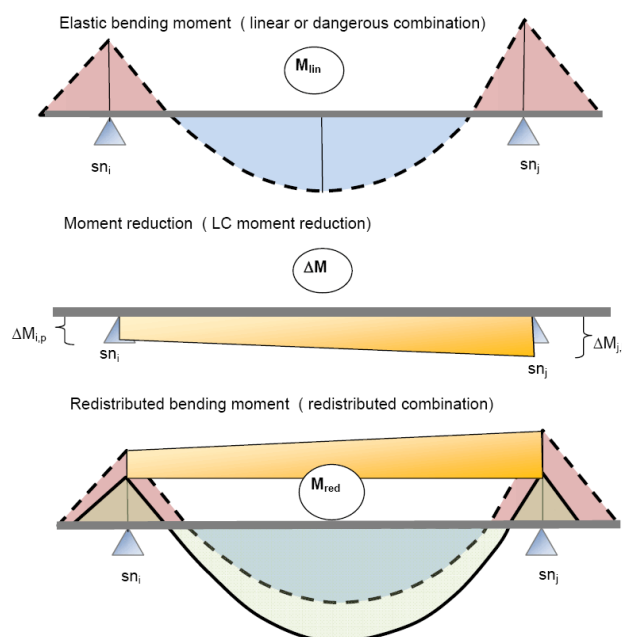
Linear calculation

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Section 5 Structural Analysis

(b) Linear elastic analysis with limited redistribution

Principle of redistribution of bending moment



Application

For analysis of structural members for the verification of ULS.

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Section 5 Structural Analysis

(b) Linear elastic analysis with limited redistribution

- The moments at ULS calculated using a linear elastic analysis may be redistributed, provided that the resulting distribution of moments remains in **equilibrium** with the applied loads.
- Redistribution of bending moments, without explicit check on the rotation capacity, is allowed in continuous beams or slabs provided that:
 - they are predominantly subject to flexure
 - the ratio of the lengths of adjacent spans is in the range of 0,5 to 2
 - $\delta \geq k_1 + k_2 x_u/d$ for $f_{ck} \leq 50$ MPa (5.10a)
 - $\delta \geq k_3 + k_4 x_u/d$ for $f_{ck} > 50$ MPa (5.10b)
 - $\geq k_5$ where Class B and Class C reinforcement is used (see EN Annex C)
 - $\geq k_6$ where Class A reinforcement is used (see EN Annex C)

Section 5 Structural Analysis

(b) Linear elastic analysis with limited redistribution

where:

δ is the ratio of the redistributed moment to the elastic bending moment

x_u is the depth of the neutral axis at the ultimate limit state after redistribution

d is the effective depth of the section

k_1, k_2, k_3, k_4, k_5 and k_6 : see recommended values in National Annex

- Redistribution should not be carried out in circumstances where the rotation capacity cannot be defined with confidence.
- For the design of columns the elastic moments from frame action should be used without any redistribution.

Section 5 Structural Analysis

(c) Plastic analysis

Plastic analysis for beams, frames and slabs

- Only suitable for ULS
- Plastic analysis without any direct check of rotation capacity may be used, if the ductility of the critical sections is sufficient for the envisaged mechanism to be formed.
- The required ductility may be deemed to be satisfied without explicit verification if all the following are fulfilled:
 - the area of tensile reinforcement is limited such that, at any section
$$x_u/d \leq 0,25 \text{ for concrete strength classes } \leq C50/60$$
$$x_u/d \leq 0,15 \text{ for concrete strength classes } \geq C55/67$$
 - reinforcing steel is either Class B or C (see EN Annex C)
 - the ratio of the moments at intermediate supports to the moments in the span is between 0,5 and 2

Section 5 Structural Analysis

(c) Plastic analysis

Options for redistribution of bending moments & plastic analysis in Scia Engineer

Available since Scia Engineer 2010.0

Choice for desired method in the Concrete Setup:

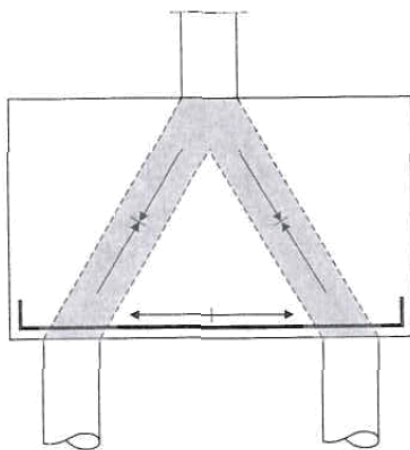
<input checked="" type="checkbox"/> Check redistributed moments	
Check acc to 5.5(4)	<input type="checkbox"/> no
Check acc to 5.6.2(2)	<input type="checkbox"/> no
Check rotation capacity 5.6.3	<input type="checkbox"/> no

Analysis with strut-and-tie models

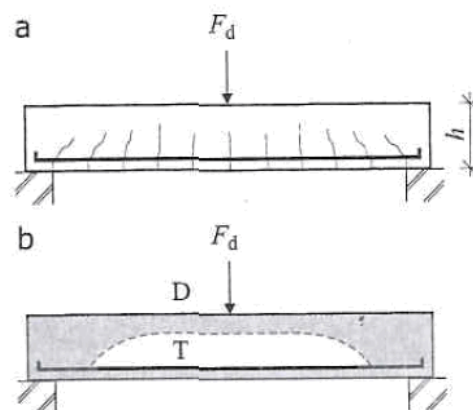
- Strut-and-tie models consist of
 - struts = compressive stress fields
 - ties = reinforcement
 - connecting nodes

- Forces: determined by maintaining the equilibrium with the applied loads in the ULS

Analysis with strut and tie models: Comparison deep beam ↔ slender beam



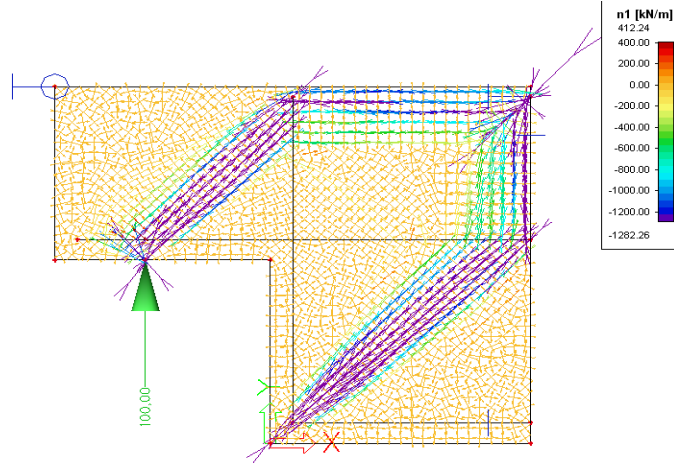
Deep beam
with pressure diagonals & tension tie



Slender beam
with pressure arch (D) & tension tie (T)

Analysis with strut and tie models in Scia Engineer

Pressure only 2D members – Trajectories result

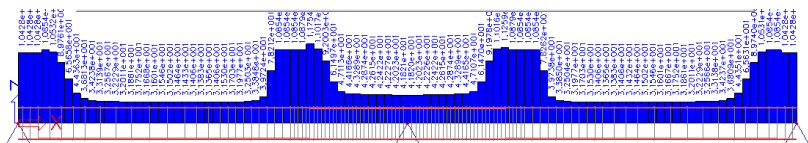


- Suitable for both ULS and SLS, provided that equilibrium and compatibility are satisfied
- Non-linear behaviour for materials taken into account
See EN Section 3 for the non-linear σ - ϵ diagrams
- The analysis may be first or second order

Non-linear analysis in Scia Engineer

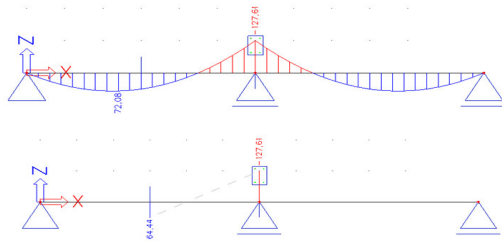
1st order: PNL calculation

2nd order: PGNL calculation

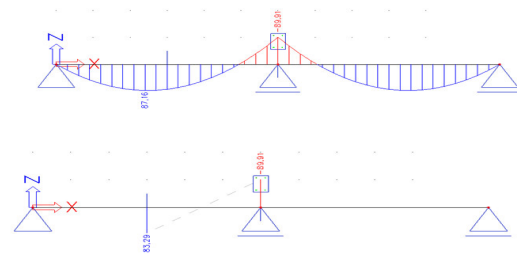


Non-linear analysis in Scia Engineer: Plastic hinges

Linear calculation



Non-linear calculation



Change of the stiffness in the concrete cross-section above the middle support due to cracks. The **plastic resistance moment** is reached.

→ Redistribution of the moment line to satisfy the equilibrium of the structure

Principle: Non-linear moment above the support + $\frac{1}{2}$ Non-linear moment at the half of the span = Linear moment above the support

Definition of Geometric imperfections

- As consideration of the unfavourable effects of possible deviations in the geometry of the structure and the position of loads.

(Deviations in cross-section dimensions → material safety factors)

- To be taken into account both in 1st and 2nd order calculation.
- To be taken into account only in ULS (persistent and accidental design situations).
- To be taken into account only in the direction where they will have the most unfavourable effect.

General

Imperfections may be represented by an **inclination θ_i** :

$$\theta_i = \theta_0 \alpha_h \alpha_m \quad (5.1)$$

where:

θ_0 is the basic value = 1/200 (recommended value)

α_h is the reduction factor for length or height

α_m is the reduction factor for number of vertical members contributing to the total effect

Isolated members

2 alternative ways to take geometric imperfections into account

(1) As an **eccentricity, e_i** , given by

$$e_i = \theta_i l_0 / 2 \quad (5.2)$$

where l_0 is the effective length

For walls and isolated columns in braced systems, $e_i = l_0 / 400$ may always be used as a simplification, corresponding to $\alpha_h = 1$.

(2) As a **transverse force, H_i** , in the position that gives the maximum moment:

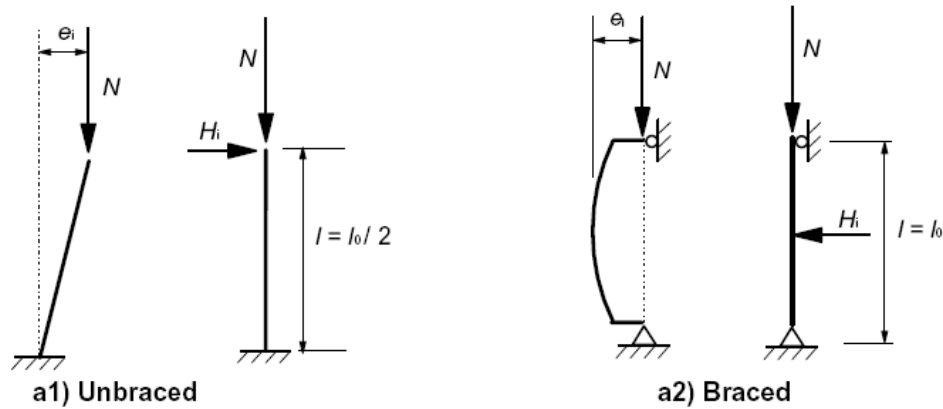
- for unbraced members: $H_i = \theta_i N$ (5.3a)

- for braced members: $H_i = 2 \theta_i N$ (5.3b)

where N is the axial load

Isolated members

2 alternative ways to take geometric imperfections into account



Minimum eccentricity for cross-section design

$$e_{0,min} = \max \{ h/30 ; 20 \text{ mm} \}$$

see EN § 6.1(4)

where h is the depth of the section

This means
$$e_0 = \max \left[(e_1 + e_i); \frac{h}{30}; 20\text{mm} \right]$$

where:

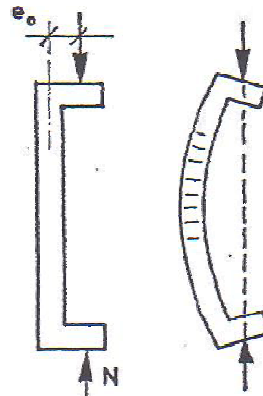
e_1 = 1st order eccentricity

e_i = eccentricity due to geometric imperfections

$e_0 = e_1 + e_i$, design eccentricity in a 1st order calculation

Definitions

- *First order effects*: action effects calculated without consideration of the effect of structural deformations, but including geometric imperfections.
- *Second order effects*: additional action effects caused by structural deformations. They shall be taken into account where they are likely to affect the overall stability of a structure significantly; e.g. in case of columns, walls, piles, arches and shells.



Criteria for 1st or 2nd order calculation

2nd order effects have to be taken into account in each direction, unless they may be ignored acc. to *one* of the following articles:

(a) 2nd order effects may be ignored if they are less than 10 % of the corresponding 1st order effects.

(b) Simplified criterion = **Slenderness criterion** for isolated members

2nd order effects may be ignored if the slenderness $\lambda < \lambda_{lim}$

In case of biaxial bending:

the slenderness criterion may be checked separately for each direction

Section 5 Structural Analysis

Analysis of second order effects with axial load

Criteria for 1st or 2nd order calculation

Slenderness ratio λ

$$\lambda = l_0 / i \quad (5.14)$$

where:

l_0 is the effective length

i is the radius of gyration of the uncracked concrete section

Limit slenderness λ_{lim}

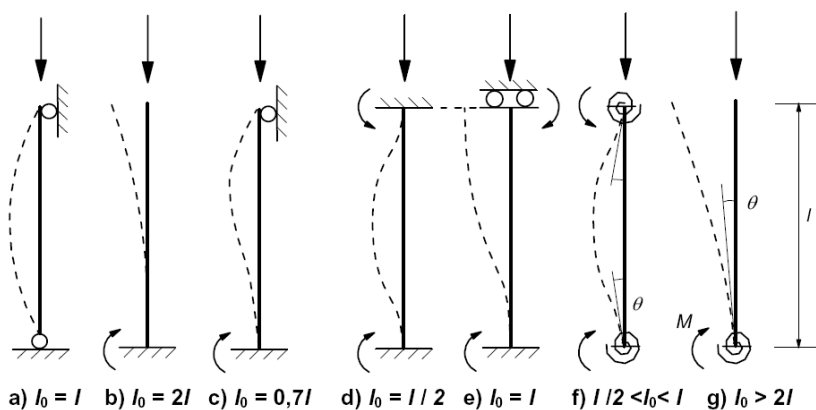
$$\lambda_{lim} = 20 A B C / \sqrt{n} \quad (\text{recommended value, 5.13N})$$

Section 5 Structural Analysis

Analysis of second order effects with axial load

Criteria for 1st or 2nd order calculation

Effective length - for isolated members



Section 5 Structural Analysis

Analysis of second order effects with axial load



Methods of analysis - Taking 2nd order effects into account

- General method: based on non-linear 2nd order analysis
- Two simplified methods: based on linear (nominal 2nd order) analysis
 - (a) Method based on nominal stiffness & moment magnification factor
 - Use: both isolated members and whole structures
 - (b) Method based on nominal curvature
 - Use: mainly suitable for isolated members, but with realistic assumptions concerning the distribution of curvature, also for structures
- The selection of simplified method (a) and (b) to be used in a Country may be found in its National Annex.

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Section 5 Structural Analysis

Analysis of second order effects with axial load



General method

General method in Scia Engineer

Real (physical and) geometrical non-linear calculation

- PGNL analysis for 1D members:
 - non-linear σ - ϵ diagram, new stiffness EI is calculated iteratively
- GNL analysis for 2D members:
 - no non-linear σ - ϵ diagram,
 - but approximation of new stiffness EI by adapting value of E in the material library:

$$EI = K_c E_{cd} I_c + K_s E_s I_s \quad \text{OR} \quad E_{cd, \text{eff}} = \frac{E_{cd}}{1 + \varphi_{ef}}$$

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Section 5 Structural Analysis

Analysis of second order effects with axial load

Simplified methods

(a) Method based on nominal stiffness

Nominal stiffness

$$EI = K_c E_{cd} I_c + K_s E_s I_s \quad E_{cd,eff} = \frac{E_{cd}}{1 + \varphi_{ef}}$$

Moment magnification factor

Total moment = 1st and 2nd order moment

$$M_{Ed} = M_{0Ed} \left[1 + \frac{\beta}{(N_B/N_{Ed}) - 1} \right]$$

Section 5 Structural Analysis

Analysis of second order effects with axial load

(b) Method based on nominal curvature

Application

For isolated members with constant normal force N and a defined effective length l_0 .

The method gives a nominal 2nd order moment based on a deflection, which in turn is based on the effective length and an estimated maximum curvature.

Design moment M_{Ed}

$$M_{Ed} = M_{0Ed} + M_2 \quad (5.31)$$

where:

M_{0Ed} is the 1st order moment, including the effect of imperfections

M_2 is the nominal 2nd order moment, based on 1st order internal forces

(b) Method based on nominal curvature

Nominal 2nd order moment M_2

$$M_2 = N_{Ed} e_2 \quad (5.33)$$

where:

N_{Ed} is the design value of axial force

e_2 is the deflection = $(1/r) * (l_0^2/c)$

$1/r$ is the curvature

l_0 is the effective length

c is a factor depending on the curvature distribution

For constant cross-section, $c = 10 (\approx \pi^2)$ is normally used. If the first order moment is constant, a lower value should be considered (8 is a lower limit, corresponding to constant total moment).

(b) Method based on nominal curvature

Curvature $1/r$

$$1/r = K_r K_\varphi 1/r_0 \quad (5.34)$$

where:

K_r is a correction factor depending on axial load

K_φ is a factor for taking account of creep

$1/r_0 = \varepsilon_{yd} / (0,45 d)$

$\varepsilon_{yd} = f_{yd} / E_s$

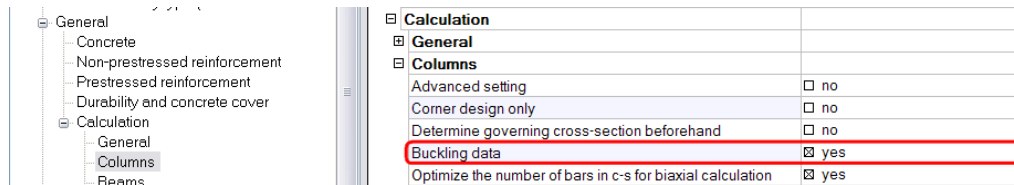
d is the effective depth

Section 5 Structural Analysis

Analysis of second order effects with axial load

(b) Method based on nominal curvature in Scia Engineer

Linear calculation



Taken into account for design:

“Buckling data” OFF: - 1st order moment

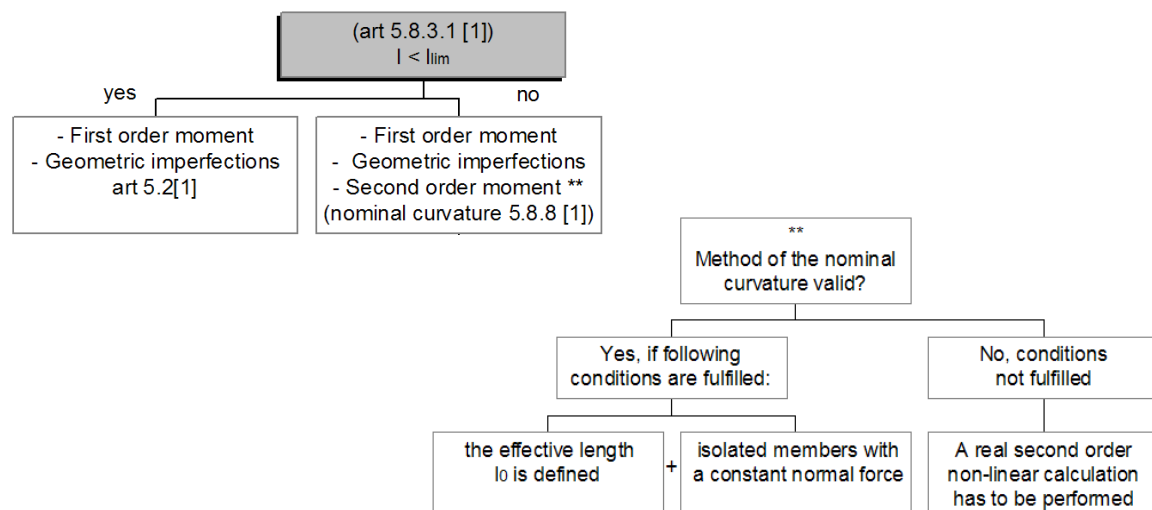
“Buckling data” ON: - 1st order moment
 - moment caused by geometrical imperfections
 - nominal 2nd order moment, only if $\lambda > \lambda_{lim}$

Section 5 Structural Analysis

Analysis of second order effects with axial load

(b) Method based on nominal curvature in Scia Engineer

Overview



Section 5 Structural Analysis

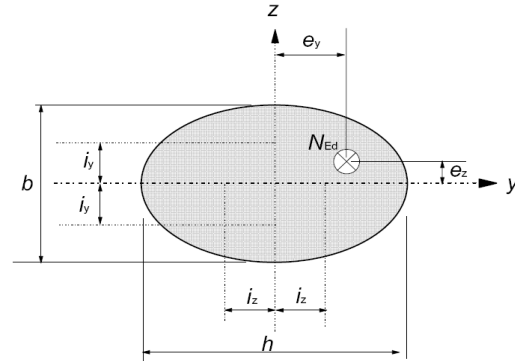
Analysis of second order effects with axial load

Bi-axial bending

To decide if a bi-axial bending calculation is required or not, the following conditions should be checked: (slenderness ratios & relative eccentricities)

$$\frac{\lambda_y}{\lambda_z} \leq 2 \quad \text{and} \quad \frac{\lambda_z}{\lambda_y} \leq 2$$

$$\frac{e_{Ed,y} / b_{eq}}{e_{Ed,z} / h_{eq}} \leq 0,2 \quad \text{or} \quad \frac{e_{Ed,z} / h_{eq}}{e_{Ed,y} / b_{eq}} \leq 0,2$$



If these conditions are NOT fulfilled → bi-axial bending calculation is required

Section 5 Structural Analysis

Analysis of second order effects with axial load

Biaxial bending

Simplified criterion = Interaction formula

$$\left(\frac{M_{Edz}}{M_{Rdz}} \right)^a + \left(\frac{M_{Edy}}{M_{Rdy}} \right)^a \leq 1 \quad (5.39)$$

where:

$M_{Edz/y}$ is the design moment around the respective axis, including a 2nd order moment (if required)

$M_{Rdz/y}$ is the moment resistance in the respective direction

a is the exponent; for circular and elliptical cross sections: a = 2;

for rectangular cross sections:

N_{Ed}/N_{Rd}	0,1	0,7	1,0
a =	1,0	1,5	2,0

with linear interpolation for intermediate values

Column calculation in Scia Engineer

Calculation Method	
Type of calculation method	Automatic determination
Automatic determination - Uni-axial b...	Uni-axial bending calculation (sum)
Design reinforcement by using (...)	Uni-axial bending calculation (max)
Area of reinforcement type	Bi-axial bending calculation (interaction formula)
Delta area of reinforcement [mm^2]	Automatic determination

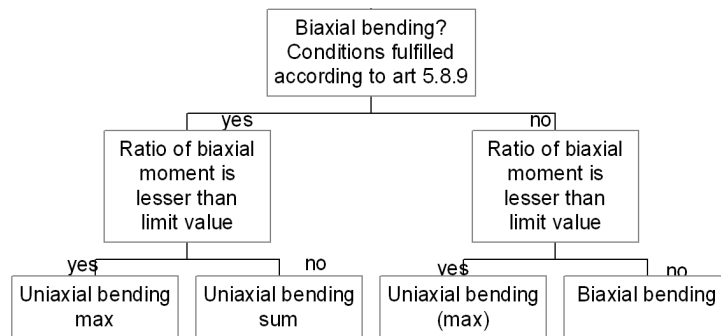
Conditions 5.8.9(3)

$$\frac{\lambda_y}{\lambda_z} \leq 2 \quad \text{and} \quad \frac{\lambda_z}{\lambda_y} \leq 2$$

$$\frac{e_{Ed,y} / b_{eq}}{e_{Ed,z} / h_{eq}} \leq 0,2 \quad \text{or} \quad \frac{e_{Ed,z} / h_{eq}}{e_{Ed,y} / b_{eq}} \leq 0,2$$

Ratio of biaxial moments

$$\frac{\min(|M_{Ed,y}|, |M_{Ed,z}|)}{\max(|M_{Ed,y}|, |M_{Ed,z}|)} \cdot 100 \leq 10\%$$



Section 6 Ultimate limit states (ULS)

Section 6 Ultimate limit states (ULS)

Bending with or without axial force



Application

Section 6 applies to undisturbed regions of beams, slabs and similar types of members for which sections remain approximately plane before and after loading.

If plane sections do not remain plane → see EN § 6.5 (Design with strut and tie models)

Ultimate moment resistance M_{Rd} (or M_u) of reinforced concrete cross-sections

Assumptions when determining M_{Rd} :

- plane sections remain plane
- strain in bonded reinforcement = strain in the surrounding concrete
- tensile strength of concrete is ignored
- stresses in concrete in compression → see design σ - ϵ diagrams (EN § 3.1.7)
- stresses in reinforcing steel → see design σ - ϵ diagrams (EN § 3.2.7)

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Section 6 Ultimate limit states (ULS)

Bending with or without axial force



Strain limits

- Reinforcing steel Tensile strain limit = ϵ_{ud} (where applicable)

- Concrete

- In sections mainly subjected to bending

Compressive strain limit = ϵ_{cu2} (or ϵ_{cu3})

- In sections subjected to \pm pure compression

(\pm concentric loading ($e_d/h < 0,1$), e.g. compression flanges of box girders, columns, ...)

Pure compressive strain limit = ϵ_{c2} (or ϵ_{c3})

For concentrically loaded cross-sections with symmetrical reinforcement, assume as

eccentricity $e_0 = \max\left[(e_1 + e_i); \frac{h}{30}; 20\text{mm}\right]$ (M_{Ed} is at least = $e_0 N_{Ed}$)

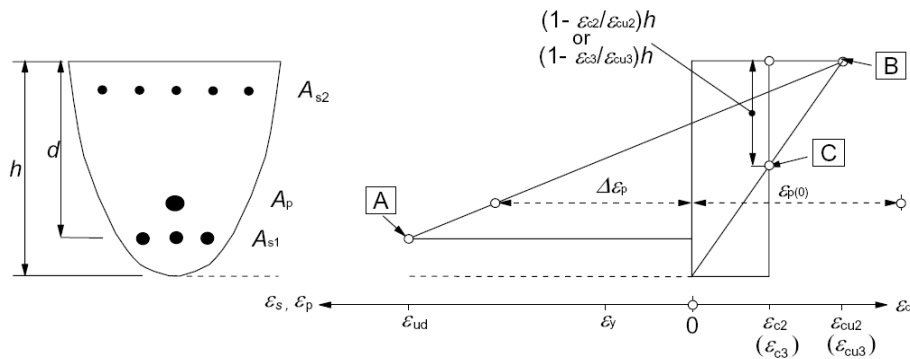
where h is the depth of the section

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Section 6 Ultimate limit states (ULS)

Bending with or without axial force

Possible range of strain distributions (ULS)



A - reinforcing steel tension strain limit

B - concrete compression strain limit

C - concrete pure compression strain limit

Section 6 Ultimate limit states (ULS)

Shear

General verification procedure

Definitions

V_{Ed} = design shear force resulting from external loading

$V_{Rd,c}$ = design shear resistance of the member *without* shear reinforcement

$V_{Rd,s}$ = design value of the shear force which can be sustained by the yielding shear reinforcement

$V_{Rd,max}$ = design value of the maximum shear force which can be sustained by the member, limited by crushing of the concrete compression struts

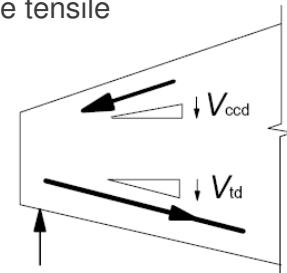
General verification procedure

Definitions

In members with inclined chords the following additional values are defined:

V_{ccd} = design value of the shear component of the force in the compression area, in the case of an inclined compression chord

V_{td} = design value of the shear component of the force in the tensile reinforcement, in the case of an inclined tensile chord



Shear resistance of a member with shear reinforcement:

$$V_{Rd} = V_{Rd,s} + V_{ccd} + V_{td} \quad (6.1)$$

General verification procedure

Overview

If $V_{Ed} \leq V_{Rd,c}$ No shear reinforcement required (theoretically), but minimum shear reinforcement should be provided for beams:

$$\rho_{w,min} = (0,08 \sqrt{f_{ck}}) / f_{yk} \quad (\text{recommended value, } 9.5N)$$

If $V_{Ed} > V_{Rd,c}$ Shear reinforcement should be provided in order that $V_{Ed} \leq V_{Rd}$

$$\text{In practice: } V_{Rd,s} = V_{Ed} - V_{ccd} - V_{td}$$

$$\text{Check if } V_{Rd,s} \text{ (or } V_{Ed}) \leq V_{Rd,max}$$

(If $V_{Ed} > V_{Rd,max}$, failure by crushing of concrete compression struts!)

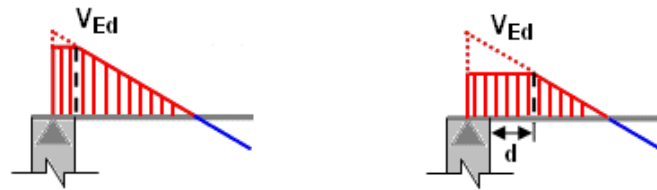
Shear force reduction at supports

For members subject to predominantly uniformly distributed loading, the design shear force need not to be checked at a distance less than d from the face of the support. Any shear reinforcement required should continue to the support.

... in Scia Engineer

<input checked="" type="checkbox"/> Beams	
Calculate compression reinforcement	<input checked="" type="checkbox"/> yes
Include normal force to calculation	<input checked="" type="checkbox"/> yes
Check compression of member	<input type="checkbox"/> no
$N_{Ed} < x \cdot A_c \cdot f_{cd}$; $x = [-]$	0,10
Moment reduction at supports	<input type="checkbox"/> no
Shear force reduction at supports	<input checked="" type="checkbox"/> yes
<input checked="" type="checkbox"/> Reduce shear force	
Reduce shear force	In the face (support/column)
<input checked="" type="checkbox"/> 2D structures	
2D structures	In the face (support/column)
<input checked="" type="checkbox"/> ULS	
ULS	In the face (support/column)+ effective height of the beam

2 options:



$V_{Ed} \leq V_{Rd,c}$: Members not requiring design shear reinforcement

Design value for the shear resistance $V_{Rd,c}$

$$V_{Rd,c} = [C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}] b_w d \quad (6.2a)$$

with a minimum of $V_{Rd,c} = (v_{min} + k_1 \sigma_{cp}) b_w d \quad (6.2b)$

where:

$$k = 1 + \sqrt{(200/d)} \leq 2,0 \text{ with } d \text{ in [mm]}$$

$\rho_l = A_{sl} / b_w d \leq 0,02$ is the longitudinal reinforcement ratio

A_{sl} is the area of the tensile reinforcement, which extends $\geq (l_{bd} + d)$ beyond the section considered

b_w is the smallest width of the cross-section in the tensile area [mm]

$\sigma_{cp} = N_{Ed} / A_c < 0,2 f_{cd}$ [MPa] with $N_{Ed} > 0$ for compression

$$C_{Rd,c} = 0,18 / \gamma_c \quad (\text{recommended value})$$

$$v_{min} = 0,035 k^{3/2} \cdot f_{ck}^{1/2} \quad (\text{recommended value, 6.3N})$$

$$k_1 = 0,15 \quad (\text{recommended value})$$

Section 6 Ultimate limit states (ULS)

Shear

$V_{Ed} \leq V_{Rd,c}$: Members not requiring design shear reinforcement

V_{Ed} should always satisfy the condition

$$V_{Ed} \leq 0,5 b_w d v f_{cd} \quad (6.5)$$

where:

v is a strength reduction factor for concrete cracked in shear

$$v = 0,6 [1 - (f_{ck}/250)] \quad (\text{recommended value, } 6.6N)$$

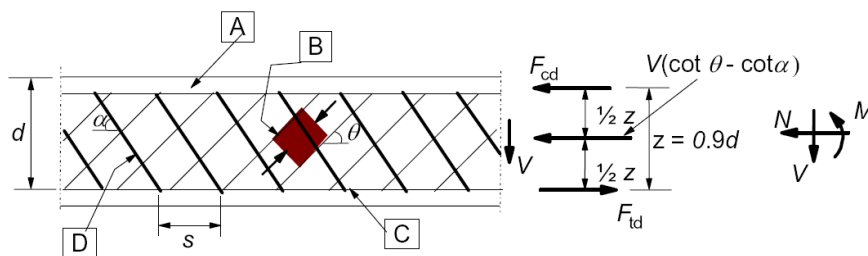
with f_{ck} in [MPa]

Section 6 Ultimate limit states (ULS)

Shear

$V_{Ed} > V_{Rd,c}$: Members requiring design shear reinforcement

The design of members with shear reinforcement is based on a **truss model**



[A] - compression chord, [B] - struts, [C] - tensile chord, [D] - shear reinforcement

α = angle between the shear reinforcement and the beam axis

θ = angle between the concrete compression strut and the beam axis

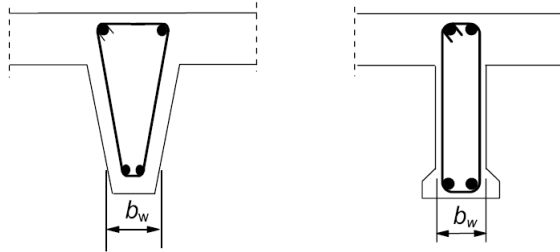
$$1 \leq \cot\theta \leq 2,5 \quad (\text{recommended limits, } 6.7N)$$

F_{td} = design value of the tensile force in the longitudinal reinforcement

F_{cd} = design value of the concrete compression force

z = the inner lever arm; the approximate value $z = 0,9 d$ may normally be used

$V_{Ed} > V_{Rd,c}$: Members requiring design shear reinforcement



b_w = minimum width between tension and compression chords

$V_{Ed} > V_{Rd,c}$: Members requiring design shear reinforcement

$\alpha = 90^\circ$ (vertical shear reinforcement)

V_{Rd} is the smaller value of:

$$V_{Rd,s} = (A_{sw}/s) z f_{ywd} \cot\theta \quad (6.8)$$

and
$$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} / (\cot\theta + \tan\theta) \quad (6.9)$$

where:

A_{sw} is the cross-sectional area of the shear reinforcement

s is the spacing of the stirrups

f_{ywd} is the design yield strength of the shear reinforcement

ν_1 is a strength reduction factor for concrete cracked in shear

α_{cw} is a coefficient taking account of the state of the stress in the compression chord

$V_{Ed} > V_{Rd,c}$: Members requiring design shear reinforcement

$\alpha = 90^\circ$ (vertical shear reinforcement)

$$v_1 = v \quad (\text{recommended value, } 6.6N)$$

If $\sigma_{wd} < 0,80 f_{ywk}$, v_1 may be taken as:

$$v_1 = 0,6 \quad \text{for } f_{ck} \leq 60 \text{ MPa} \quad (6.10.aN)$$

$$v_1 = 0,9 - f_{ck} / 200 > 0,5 \quad \text{for } f_{ck} \geq 60 \text{ MPa} \quad (6.10.bN)$$

If this Expression (6.10) is used, the value of f_{ywd} should be reduced to $0,80 f_{ywk}$ in Expression (6.8).

$$\alpha_{cw} = 1 \quad \text{for non-prestressed structures} \quad (\text{recommended value})$$

$A_{sw,max}$, for $\theta = 45^\circ$ ($\cot\theta = 1$ and $\tan\theta = 1$), and $V_{Rd,s} = V_{Rd,max}$

$$A_{sw,max}/s = 0,5 \alpha_{cw} b_w v_1 f_{cd} / f_{ywd} \quad (6.12)$$

$V_{Ed} > V_{Rd,c}$: Members requiring design shear reinforcement

$\alpha < 90^\circ$ (inclined shear reinforcement)

V_{Rd} is the smaller value of:

$$V_{Rd,s} = (A_{sw}/s) z f_{ywd} (\cot\theta + \cot\alpha) \sin\alpha \quad (6.13)$$

$$\text{and } V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{cd} (\cot\theta + \cot\alpha) / (1 + \cot^2\theta) \quad (6.14)$$

$A_{sw,max}$, for $\theta = 45^\circ$ ($\cot\theta = 1$), and $V_{Rd,s} = V_{Rd,max}$

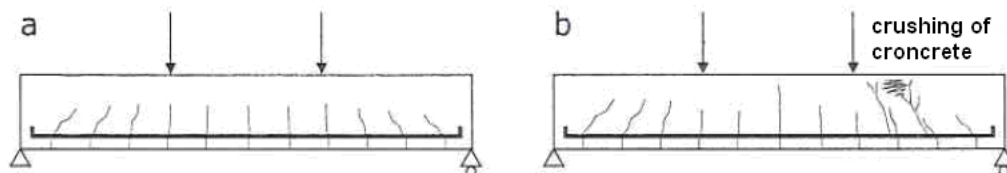
$$A_{sw,max}/s = (0,5 \alpha_{cw} b_w v_1 f_{cd}) / (f_{ywd} \sin\alpha) \quad (6.15)$$

Section 6 Ultimate limit states (ULS)

Shear

Explanation for $V_{Rd,max}$ – Failure modes in case of shear

(1) Shear-bending failure



a) pattern of cracking in the SLS

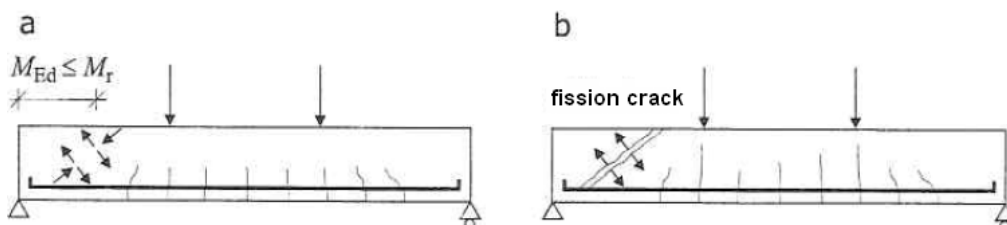
b) pattern of cracking in the ULS

Section 6 Ultimate limit states (ULS)

Shear

Explanation for $V_{Rd,max}$ – Failure modes in case of shear

(2) Shear-tension failure



a) pattern of cracking in the SLS

b) pattern of cracking in the ULS

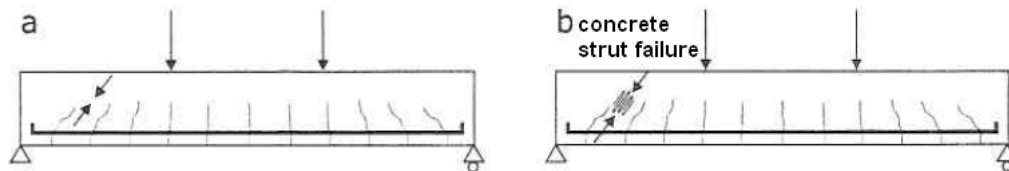
→ Adding enough shear reinforcement (in the form of vertical stirrups) prevents shear-bending and shear-tension failure.

Section 6 Ultimate limit states (ULS)

Shear

Explanation for $V_{Rd,max}$ – Failure modes in case of shear

(3) Shear-compression failure



a) pattern of cracking in the SLS

b) pattern of cracking in the ULS

→ Imposing a maximum value $V_{Rd,max}$ to the shear force V_{Ed} prevents sudden failure of the concrete compression strut before yielding of the shear reinforcement.

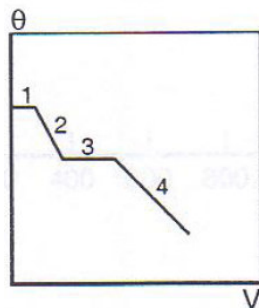
Section 6 Ultimate limit states (ULS)

Shear

Variable strut inclination method

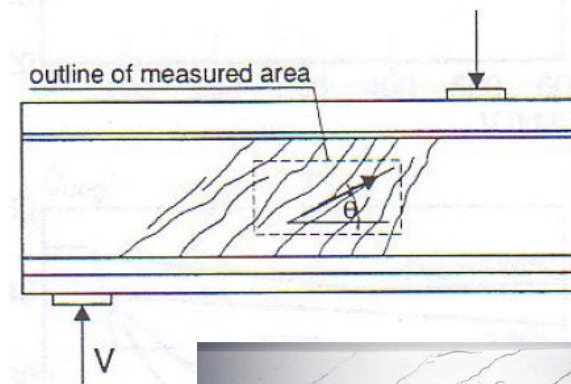
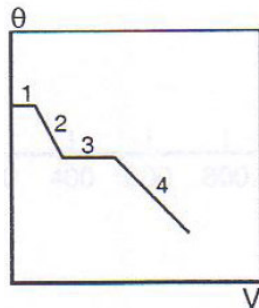
The strut inclination θ may be chosen between two limit values

$$1 \leq \cot\theta \leq 2,5 \quad \text{or} \quad 21,8^\circ < \theta < 45^\circ \quad (\text{recommended limits, 6.7N})$$



- 1) Web uncracked in shear
- 2) Inclined cracks appear
- 3) Stabilization of inclined cracks
- 4) Yielding of stirrups → further rotations and new cracks under lower angle → finally failure by web crushing

Variable strut inclination method



Variable strut inclination method

Advantages of this method:

Large freedom of design because of the large interval for θ

By making a good choice for the inclination of the struts, optimal design can be achieved:

- Larger angle θ → higher value of $V_{Rd,max}$ (saving on concrete)
- Smaller angle θ → larger stirrup spacing is sufficient = smaller value of A_{sw} (saving on steel)

Section 6 Ultimate limit states (ULS)

Shear

Variable strut inclination method

General procedure – which can be used in Scia Engineer:

- Assume $\theta = 21,8^\circ$ ($\cot \theta = 2,5$) and calculate A_{sw}
- Check if $V_{Ed} > V_{Rd,max}$:
 - If NO → OK, end of design
 - If YES → crushing of the concrete strut
- 3 options if $V_{Rd,max}$ is exceeded:
 - increase height of beam
 - choose higher concrete class
 - increase θ , or calculate θ for which $V_{Ed} = V_{Rd,s}$

and repeat the procedure

Section 6 Ultimate limit states (ULS)

Shear

Variable strut inclination method

User input of angle θ (or cotangent θ) in Scia Engineer

☐ Shear	
☐ 1D structures	
☐ Shear coefficients	
Distance with full resistance from outside stirrup (multiple ...	0,50
☐ Angle between the concrete compression strut a...	
Type of input theta	Angle
☐ Web	Angle
theta [deg]	Cotangent
cot(theta)	1,192
☐ Compression flange	
theta [deg]	40,00
cot(theta)	1,192
☐ Tension flange	
theta [deg]	40,00
cot(theta)	1,192

Additional tensile force in the longitudinal reinforcement ... caused by shear

2 approaches

(1) EN Section 6:

→ For members with shear reinforcement

Calculation of the **additional tensile force**, ΔF_{td} , in the longitudinal reinforcement due to shear V_{Ed} :

$$\Delta F_{td} = 0,5 V_{Ed} (\cot\theta - \cot\alpha) \quad (6.18)$$

$(M_{Ed}/z) + \Delta F_{td} \leq M_{Ed,max}/z$, where $M_{Ed,max}$ is the maximum moment along the beam

Additional tensile force in the longitudinal reinforcement ... caused by shear

(2) EN Section 9:

→ For members without shear reinforcement

ΔF_{td} may be estimated by **shifting the moment curve** (in the region cracked in flexure) a distance $a_1 = d$ in the unfavourable direction .

→ For members with shear reinforcement

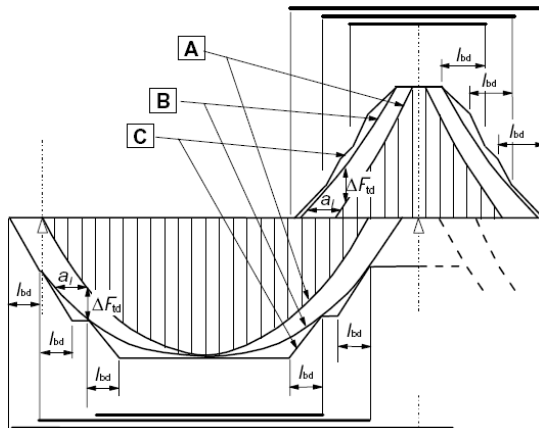
This "shift rule" may also be used as an alternative to approach (1), where:

$$a_1 = z (\cot\theta - \cot\alpha) / 2 \quad (9.2)$$

Section 6 Ultimate limit states (ULS)

Shear

Additional tensile force in the longitudinal reinforcement ... caused by shear



[A] - Envelope of $M_{Ed}/z + N_{Ed}$ [B] - acting tensile force F_s [C] - resisting tensile force F_{Rs}

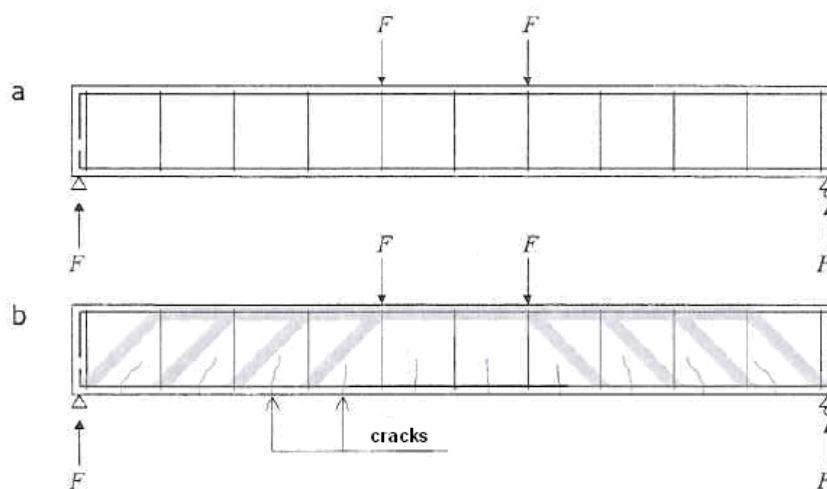
The curtailment of longitudinal reinforcement, taking into account the effect of inclined cracks and the resistance of reinforcement bars within their anchorage lengths.

(As a conservative simplification the contribution of the anchorage may be ignored.)

Section 6 Ultimate limit states (ULS)

Shear

Explanation for the additional tensile force

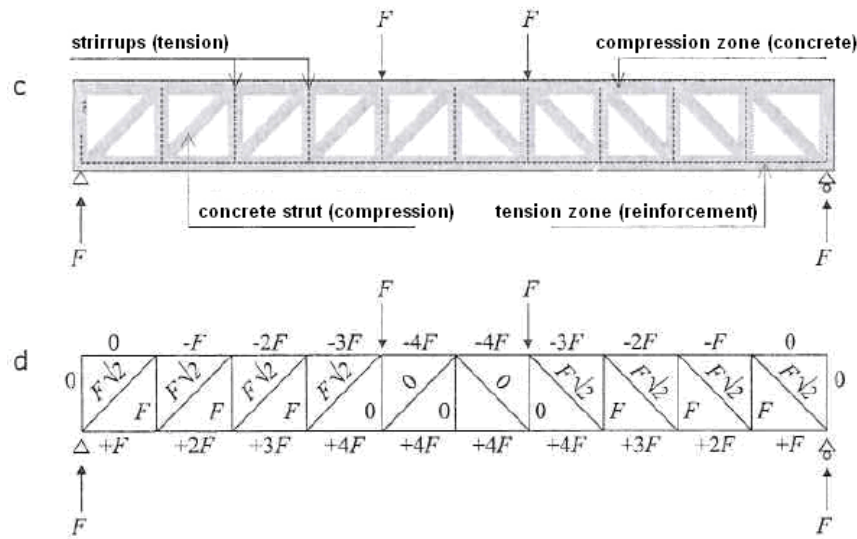


a) reinforced concrete beam b) cracked beam with compression struts, after loading

Section 6 Ultimate limit states (ULS)

Shear

Explanation for the additional tensile force



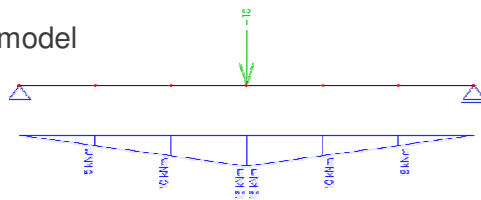
c) truss analogy ($\theta = 45^\circ$) d) internal forces in the members of the truss

Section 6 Ultimate limit states (ULS)

Shear

Explanation for the additional tensile force

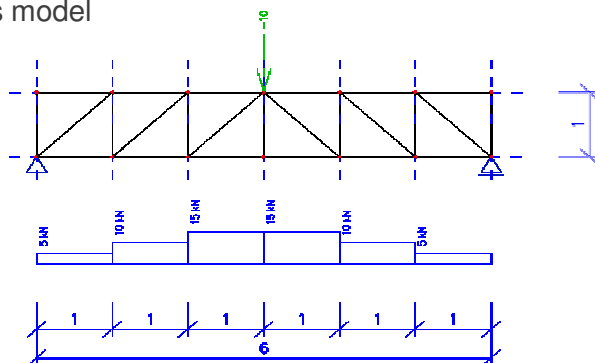
- Beam model



$$N = M / z$$

assume $z = 1$

- Truss model



Conclusion:

Design of reinforcement according to beam model is unsafe \rightarrow shift of M-line

General

The torsional resistance of a section may be calculated on the basis of a **thin-walled closed section**, in which equilibrium is satisfied by a closed shear flow.

- Solid section → equivalent thin-walled section
- Complex shape (e.g. T-sections) → series of equivalent thin-walled section, where the total torsional resistance = sum of the capacities of the individual elements
- Non-solid sections → equivalent wall thickness \leq actual wall thickness

Design procedure

Definitions

The shear stress in a wall i of a section subject to a pure torsional moment:

$$\tau_{t,i} \ t_{ef,i} = T_{Ed} / 2A_k \quad (6.26)$$

The shear force $V_{Ed,i}$ in a wall i due to torsion is given by:

$$V_{Ed,i} = \tau_{t,i} \ t_{ef,i} \ Z_i \quad (6.27)$$

where

T_{Ed} is the applied design torsion

$\tau_{t,i}$ is the torsional shear stress in wall i

Section 6 Ultimate limit states (ULS)

Torsion

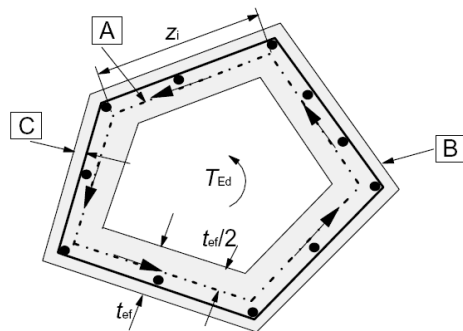
A_k is the area enclosed by the centre-lines of the connecting walls, including inner hollow areas

$t_{ef,i}$ is the effective wall thickness, which may be taken as A/u

A is the total area of the cross-section within the outer circumference, including inner hollow areas

u is the outer circumference of the cross-section

z_i is the side length of wall i



A - centre-line

B - outer edge of effective cross-section, circumference u ,

C - cover

Section 6 Ultimate limit states (ULS)

Torsion

Longitudinal reinforcement for torsion ΣA_{sl}

$$\Sigma A_{sl} = (T_{Ed} \cot \theta u_k) / (2 A_k f_{yd}) \quad (6.28)$$

where:

u_k is the perimeter of the area A_k

f_{yd} is the design yield stress of the longitudinal reinforcement A_{sl}

θ is the angle of compression struts

In compressive chords: The longitudinal reinf. may be reduced in proportion to the available compressive force.

In tensile chords: The longitudinal reinf. for torsion should be added to the other reinforcement. It should be distributed over the length of side, z_i , but for smaller sections it may be concentrated at the ends of this length.

Transverse reinforcement for torsion (and shear)

The effects of torsion (T) and shear (S) may be superimposed, assuming the same value for the strut inclination θ . Limits for θ are given in (6.7N).

This means $V_{Ed} = V_{Ed}(S) + V_{Ed}(T)$

$$\text{where: } V_{Ed}(T) = \sum V_{Ed,i} \quad (6.26) - (6.27)$$

$$\text{For each wall } i: V_{Ed,i} = \tau_{t,i} t_{ef,i} z_i = (T_{Ed} z_i) / (2 A_k) \quad (6.26) - (6.27)$$

$$\text{In practice: } V_{Ed} = V_{Rd,s} = (A_{sw}/s) z f_{ywd} \cot \theta \quad (6.8)$$

Specific conditions to be checked: Shear – Torsion interaction diagrams

1st Condition

In order not to exceed the bearing capacity of the concrete struts for a member subjected to torsion and shear, the following condition should be satisfied:

$$(T_{Ed} / T_{Rd,max}) + (V_{Ed} / V_{Rd,max}) \leq 1 \quad (6.29)$$

where:

T_{Ed} is the design torsional moment & V_{Ed} is the design transverse force

$T_{Rd,max}$ is the design torsional resistance moment

$$T_{Rd,max} = 2 v \alpha_{cw} f_{cd} A_k t_{ef,i} \sin \theta \cos \theta \quad (6.30)$$

where v follows from (6.6N) and α_{cw} from (6.9)

$V_{Rd,max}$ is the maximum design shear resistance according to (6.9) or (6.14). In solid cross-sections the full width of the web may be used to determine $V_{Rd,max}$.

Section 6 Ultimate limit states (ULS)

Torsion

Specific conditions to be checked: Shear – Torsion interaction diagrams

2nd Condition

For approximately rectangular solid sections, only minimum reinforcement is required if the following condition is satisfied:

$$(T_{Ed} / T_{Rd,c}) + (V_{Ed} / V_{Rd,c}) \leq 1 \quad (6.31)$$

where :

$T_{Rd,c}$ is the torsional cracking moment, which may be determined by setting $\tau_{t,i} = f_{ctd}$

$V_{Rd,c}$ follows from (6.2)

Minimum transverse reinforcement → see $\rho_{w,min}$ (9.5N)

Section 6 Ultimate limit states (ULS)

Torsion

Torsion in Scia Engineer

Not taken into account by default !

<input checked="" type="checkbox"/> Calculation	
<input checked="" type="checkbox"/> General	
Number of iteration steps	100
Precision of iteration [%]	1
Limit value for checks [-]	1.00
User defined and end sections only	<input type="checkbox"/> no
Concrete area weakened by reinforcement bars	<input type="checkbox"/> no
Concrete area weakened by prestressed reinforcement	<input type="checkbox"/> no
For design calculations of 1D members, consider longitud...	<input checked="" type="checkbox"/> yes
Check torsion	<input checked="" type="checkbox"/> yes
Check shear of construction joint	<input type="checkbox"/> no
Calculation of additional force caused by shear and torsion	Method according to 9.2.1.3 ▾

! Torsion reinforcement is only calculated for the walls i // local z axis of the beam !

For A_{st} : all of the required reinf. is distributed over the walls i // local z axis

For A_{sw} : only the required reinf. for $V_z(T)$ is calculated, not the one for $V_y(T)$

Section 6 Ultimate limit states (ULS)

Punching

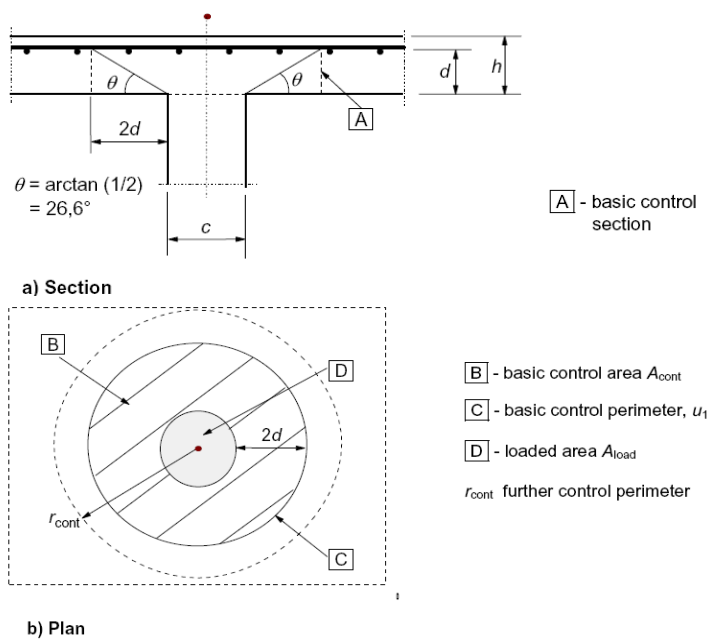
General

- Punching = 'extension' of the shear principles
- Punching shear results from a **concentrated load or reaction**, acting on a small area A_{load} (the loaded area of a slab or a foundation)
- Verification model for checking punching failure at the ULS, based on control perimeters where checks will be performed.

Section 6 Ultimate limit states (ULS)

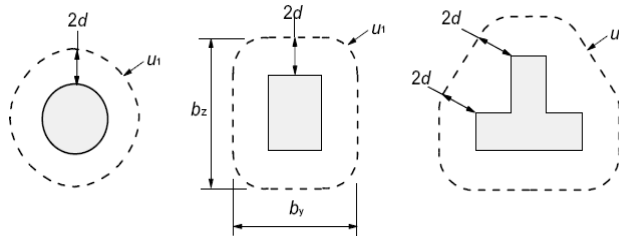
Punching

Verification model for checking punching failure at the ULS:



Basic control perimeter u_1

Normally taken at a distance $2d$ from the loaded area:



The effective depth d_{eff} of the slab is assumed constant:

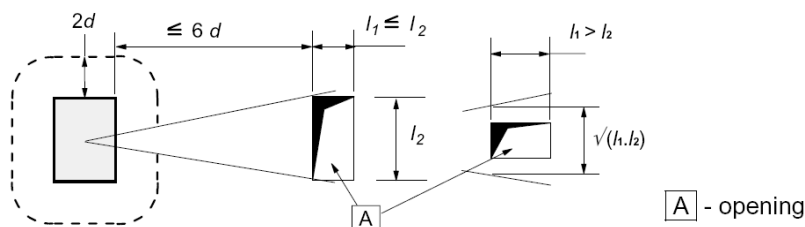
$$d_{eff} = (d_y + d_z) / 2 \tag{6.32}$$

where d_y and d_z are the effective depths of the reinf. in 2 orthogonal directions

In case the concentrated force is opposed by a high pressure (e.g. soil pressure on a column base), control perimeters at a distance less than $2d$ should be considered.

Basic control perimeter u_1

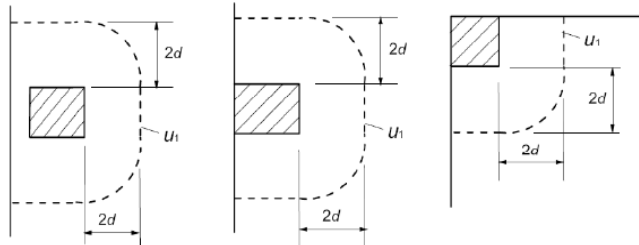
In case of a loaded area near an opening:



If the shortest distance between the perimeter of the loaded area and the edge of the opening $\le 6d$, the part of the control perimeter contained between two tangents is considered ineffective.

Basic control perimeter u_1

In case of a loaded area near an edge or corner:



If the distance to the edge or corner is smaller than d , special edge reinforcement should always be provided, see EN § 9.3.1.4.

Further perimeters u_i

Further perimeters u_i should have the same shape as the basic control perimeter u_1 .

Design procedure

Based on checks at the face of the column and at the basic control perimeter u_1 .

If shear reinforcement is required, a further perimeter $u_{out,ef}$ should be found where shear reinforcement is no longer required.

Definition of design shear resistances [MPa]

$V_{Rd,c}$ = design value of the punching shear resistance of a slab *without* punching shear reinforcement along the control section considered

$V_{Rd,cs}$ = design value of the punching shear resistance of a slab *with* punching shear reinforcement along the control section considered

$V_{Rd,max}$ = design value of the *maximum* punching shear resistance along the control section considered

Design procedure

Checks to be performed

- Check at the face of the column, or at the perimeter of the loaded area (perimeter u_0):

$$V_{Ed0} \leq V_{Rd,max}$$

with v_{Ed0} the design shear stress at the column perimeter u_0

- Check at the basic control perimeter u_1 :

If $v_{Ed} \leq v_{Rd,c}$: Punching shear reinforcement is not required

If $v_{Ed} > v_{Rd,c}$: Punching shear reinforcement has to be provided acc. to (6.52)

with v_{Ed} the design shear stress at the basic control perimeter u_1

Design procedure

Remark: Where the support reaction is eccentric with regard to the control perimeter, the maximum shear stress should be taken as:

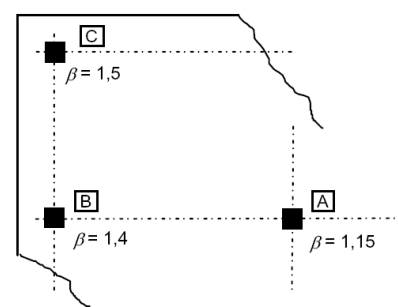
$$v_{Ed} = \beta V_{Ed} / u_i d \quad (6.38)$$

where:

β can be calculated with the formulas in EN § 6.4.3(3)-(4)-(5)

Often, approximate values for β may be used:

(recommended values for internal (A), edge (B) and corner(C) columns)



Design procedure

Remark: In case of a foundation slab, the punching shear force V_{Ed} may be reduced due to the favourable action of the soil pressure.

$$V_{Ed,red} = V_{Ed} - \Delta V_{Ed} \quad (\text{for concentric loading}) \quad (6.48)$$

where:

V_{Ed} is the applied shear force

ΔV_{Ed} is the net upward force within the control perimeter considered, i.e. upward pressure from soil minus self weight of base

$$v_{Ed} = V_{Ed,red} / u d \quad (6.49)$$

Remember: Consider control perimeters *within* $2d$ from the periphery of the column.

$V_{Ed} \leq V_{Rd,c}$: No punching shear reinforcement required

Design punching shear resistance of a slab *without* shear reinforcement $v_{Rd,c}$

$$v_{Rd,c} = C_{Rd,c} k (100 \rho_1 f_{ck})^{1/3} + k_1 \sigma_{cp} \geq (v_{min} + k_1 \sigma_{cp}) \quad (6.47)$$

where:

$$C_{Rd,c} = 0,18 / \gamma_c \quad (\text{recommended value})$$

$$v_{min} = 0,035 k^{3/2} \cdot f_{ck}^{1/2} \quad (\text{recommended value, 6.3N})$$

$$k_1 = 0,1 \quad (\text{recommended value})$$

→ Analogy with (6.2a) and (6.2b)

$V_{Ed} > V_{Rd,c}$: Punching shear reinforcement required

Design punching shear resistance of a slab *with* shear reinforcement $v_{Rd,cs}$

$$V_{Rd,cs} = 0,75 v_{Rd,c} + 1,5 (d/s_r) A_{sw} f_{ywd,ef} (1/(u_1 d)) \sin \alpha \quad (6.52)$$

where:

A_{sw} is the area of one perimeter of shear reinforcement around the column [mm²]

s_r is the radial spacing of perimeters of shear reinforcement [mm]

$f_{ywd,ef}$ is the effective design strength of the punching shear reinforcement, $f_{ywd,ef} = 250 + 0,25 d \leq f_{ywd}$ [MPa]

d is the mean of the effective depths in the orthogonal directions [mm]

α is the angle between the shear reinforcement and the plane of the slab

→ Analogy with (6.13)

$V_{Ed} > V_{Rd,c}$: Punching shear reinforcement required

Design punching shear resistance of a slab *with* shear reinforcement $v_{Rd,cs}$

$$V_{Rd,cs} = 0,75 v_{Rd,c} + 1,5 (d/s_r) A_{sw} f_{ywd,ef} (1/(u_1 d)) \sin \alpha \quad (6.52)$$

Explanation of the formula:

$$V_{Rd,cs} = 0,75 v_{Rd,c} + V_{Rd,s}$$

- The contribution of the steel comes from the shear reinforcement at $1,5 d$ from the loaded area.

- The contribution of the concrete is 75% of the resistance of a slab without punching shear reinforcement.

Section 6 Ultimate limit states (ULS)

Punching

$V_{Ed} > V_{Rd,c}$: Punching shear reinforcement required

Design maximum punching shear resistance $v_{Rd,max}$

$$v_{Ed} = \beta V_{Ed} / u_0 d \leq v_{Rd,max} \quad (6.53)$$

$$v_{Rd,max} = 0,5 v f_{cd} \quad (\text{recommended value})$$

where

u_0 for an interior column $u_0 = \text{length of column periphery [mm]}$

for an edge column $u_0 = c_2 + 3d \leq c_2 + 2c_1$ [mm]

for a corner column $u_0 = 3d \leq c_1 + c_2$ [mm]

c_1, c_2 are the column dimensions

v see (6.6)

β see EN § 6.4.3(3)-(4)-(5)-(6)

Section 6 Ultimate limit states (ULS)

Punching

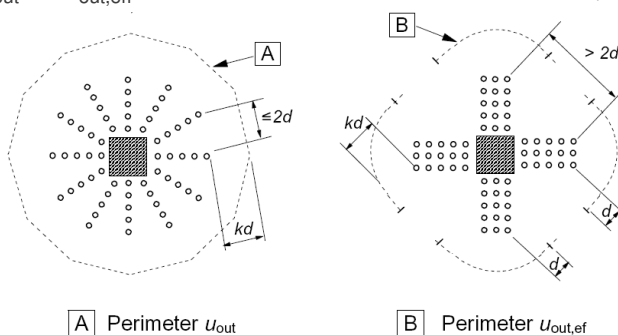
$V_{Ed} > V_{Rd,c}$: Punching shear reinforcement required

Control perimeter at which shear reinforcement is no longer required, u_{out} or $u_{out,ef}$

$$u_{out,ef} = \beta V_{Ed} / (v_{Rd,c} d) \quad (6.54)$$

The outermost perimeter of shear reinforcement should be placed at a distance $\leq k d$ within u_{out} or $u_{out,ef}$:

$k = 1,5$ (recommended value)



Section 7 Serviceability limit states (SLS)

Section 7 Serviceability limit states (SLS)

General

Common serviceability limit states

- Stress limitation
- Crack control
- Deflection control

Other limit states, like vibration, are not covered in this Standard.

Section 7 Serviceability limit states (SLS)

Stress limitation



Limitation of the compressive stress in the concrete

under the **characteristic** combination of loads

... to avoid longitudinal cracks, micro-cracks or high levels of creep, where they could result in unacceptable effects on the function of the structure

e. g. To avoid longitudinal cracks, which may lead to a reduction of durability:

Limitation of the compressive stress to a value $k_1 f_{ck}$,

in areas exposed to environments of exposure classes XD, XF and X

where $k_1 = 0,6$ (recommended value)

Other (equivalent) measures:

- an increase in the cover to reinforcement in the compressive zone
- confinement by transverse reinforcement

Section 7 Serviceability limit states (SLS)

Stress limitation



Limitation of the compressive stress in the concrete

under the **quasi-permanent** combination of loads

... to avoid non-linear creep

If $\sigma_c \leq k_2 f_{ck}$ linear creep may be assumed

If $\sigma_c > k_2 f_{ck}$ non-linear creep should be considered

where $k_2 = 0,45$ (recommended value)

Section 7 Serviceability limit states (SLS)

Stress limitation

Limitation of the tensile stress in the reinforcement

under the **characteristic** combination of loads

... to avoid inelastic strain, unacceptable cracking or deformation

e.g. To avoid unacceptable cracking or deformation:

Limitation of the tensile stress to a value $k_3 f_{yk}$

where $k_3 = 0,8$

(recommended value)

e.g. In case of an imposed deformation:

Limitation of the tensile stress to a value $k_4 f_{yk}$

where $k_4 = 1$

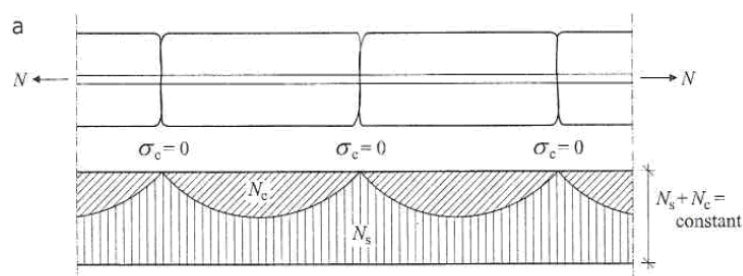
(recommended value)

Section 7 Serviceability limit states (SLS)

Crack control

Principle

Cracking of an axially loaded reinforced concrete column



a. N_c and N_s in relation to the pattern of cracks



b. Pattern of cracks in case of one large reinforcement bar



c. Pattern of cracks in case of four small reinforcement bars

Section 7 Serviceability limit states (SLS)

Crack control

Limitation of cracking

under the **quasi-permanent** combination of loads

... to guarantee the proper functioning and durability of the structure, and acceptable appearance

- Cracking is normal in reinforced concrete structures!
- Cracks may be permitted to form without any attempt to control their width, provided that they do not impair the functioning of the structure.

Section 7 Serviceability limit states (SLS)

Crack control

Max. crack width w_{max}

$w_{max} =$ (recommended values, EN Table 7.1N)

Exposure Class	Reinforced members and prestressed members with unbonded tendons	Prestressed members with bonded tendons
	Quasi-permanent load combination	Frequent load combination
X0, XC1	0,4 ¹	0,2
XC2, XC3, XC4	0,3	0,2 ²
XD1, XD2, XS1, XS2, XS3		Decompression
Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.		
Note 2: For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.		

... taking into account the proposed function and nature of the structure and the costs of limiting cracking

Minimum reinforcement areas

- A minimum amount of bonded reinforcement is required to control cracking in areas where tension is expected.
- The amount may be estimated from equilibrium between the tensile force in concrete just before cracking and the tensile force in reinforcement at yielding.
(or at a lower stress if necessary to limit the crack width)

Min. reinforcement area $A_{s,min}$ (within the tensile zone)

$$A_{s,min} = (k_c k_{ct,eff} f_{ct}) / \sigma_s \quad (7.1)$$

where:

A_{ct} is the area of concrete within the tensile zone, just before the formation of the first crack

σ_s is the maximum stress permitted in the reinforcement immediately after formation of the crack: $\sigma_s = f_{yk}$, unless a lower value is needed to satisfy the crack width limits according to the maximum bar size or spacing (see further)

$f_{ct,eff} = f_{ctm}$ or $(f_{ctm}(t))$ if cracking is expected earlier than 28 days

$k = 1,0$ for webs with $h \leq 300$ mm or flanges with widths ≤ 300 mm

$= 0,65$ for webs with $h \leq 800$ mm or flanges with widths > 800 mm

k_c is a coefficient which takes account of the stress distribution within the section immediately prior to cracking and of the change of the lever arm

Section 7 Serviceability limit states (SLS)

Crack control



Two alternative methods for limitation of cracking:

- Calculation of crack widths,
to check if $w_k \leq w_{\max}$
- Control of cracking without direct calculation,
but by restricting the bar diameter or spacing (simplified method)

Section 7 Serviceability limit states (SLS)

Crack control



Control of cracking without direct calculation

Where $A_{s,\min}$ is provided, and for cracks caused mainly by loading, crack widths are unlikely to be excessive if:

either the bar diameters (EN Table 7.2N) **or** the bar spacing (EN Table 7.3N) are not exceeded.

- The steel stress should be calculated on the basis of a cracked section under the relevant combination of actions.

- The values in the tables are based on the following assumptions: $c = 25\text{mm}$; $f_{ct,\text{eff}} = 2,9\text{MPa}$; $h_{cr} = 0,5$; $(h-d) = 0,1h$; $k_1 = 0,8$; $k_2 = 0,5$; $k_c = 0,4$; $k = 1,0$; $k_t = 0,4$ and $k' = 1,0$

Section 7 Serviceability limit states (SLS)

Crack control

Control of cracking without direct calculation

Steel stress ² [MPa]	Maximum bar size [mm]		
	w _k = 0,4 mm	w _k = 0,3 mm	w _k = 0,2 mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

EN Table 7.2N

Maximum bar diameters
 ϕ_s^* for crack control

Steel stress ² [MPa]	Maximum bar spacing [mm]		
	w _k =0,4 mm	w _k =0,3 mm	w _k =0,2 mm
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-

EN Table 7.3N

Maximum bar spacing
for crack control

Section 7 Serviceability limit states (SLS)

Crack control

Calculation of crack widths w_k

$$w_k = s_{r,max} (\epsilon_{sm} - \epsilon_{cm}) \quad (7.8)$$

where

$s_{r,max}$ is the maximum crack spacing

ϵ_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening.

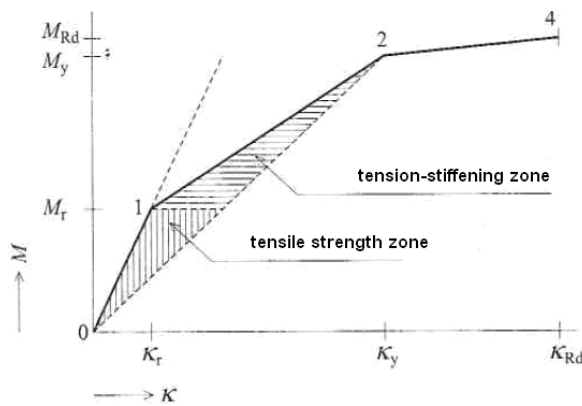
ϵ_{cm} is the mean strain in the concrete between cracks

For the formulas for $(\epsilon_{sm} - \epsilon_{cm})$ and $s_{r,max}$, see EN § 7.3.4

Section 7 Serviceability limit states (SLS)

Deflection control

Principle



M_r = moment of 1st cracking

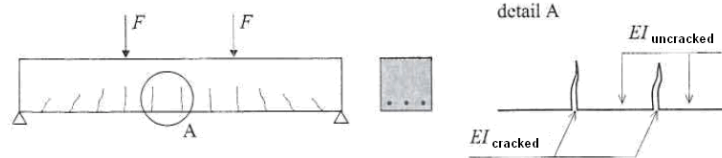
M_y = yielding moment (steel)

M_{Rd} = moment of resistance (failure of concrete under compression)

zone 0-1: no cracks

zone 1-2: cracks arise and widen

zone 2-4: cracks become visibly wide (control mechanism to failure)



Section 7 Serviceability limit states (SLS)

Deflection control

Limitation of deflection

under the **quasi-permanent** combination of loads

... to avoid adversely affection of the proper functioning or appearance

Limitation of the calculated sag of a beam, slab or cantilever:

$$1/250 * \text{span}$$

Limitation of deflections that could damage adjacent parts of the structure:

$$1/500 * \text{span} \quad (\text{deflection after construction})$$

Section 7 Serviceability limit states (SLS)

Deflection control



Two alternative methods for limitation of deflection:

- Calculation of deflection,
to check if the calculated value \leq the limit value
- Control of deflection without direct calculation,
but by limiting the span/depth ratio

Section 7 Serviceability limit states (SLS)

Deflection control



Checking deflections by calculation

- Consideration of 2 conditions
 - (I) uncracked condition
 - (II) fully cracked condition

Members which are expected to crack, but may not be fully cracked, will behave in a manner **intermediate** between the uncracked and fully cracked conditions.

Section 7 Serviceability limit states (SLS)

Deflection control



Checking deflections by calculation

- A prediction of behaviour (for members subjected mainly to flexure) is given by:

$$\alpha = \zeta \alpha_{II} + (1 - \zeta) \alpha_I \quad (7.18)$$

where:

α is the deformation parameter considered, e.g. a strain, a curvature, a rotation, or – as a simplification – a deflection

α_I, α_{II} are the values for the uncracked and fully cracked conditions

ζ is a distribution coefficient (allowing for tensioning stiffening at a section)

Section 7 Serviceability limit states (SLS)

Deflection control



Checking deflections by calculation

$$\zeta = 1 - \beta (\sigma_{sr}/\sigma_s)^2 \quad (7.19)$$

$\zeta = 0$ for uncracked sections

β is a coefficient taking account of the influence of the duration of the loading

$\beta = 1,0$ for a single short-term loading

$\beta = 0,5$ for sustained loads or many cycles of repeated loading

σ_s is the stress in the tension reinforcement calculated on the basis of a cracked section

σ_{sr} is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking

Note: σ_{sr}/σ_s may be replaced by M_{cr}/M for flexure or N_{cr}/N for pure tension, where M_{cr} is the cracking moment and N_{cr} is the cracking force.

Checking deflections by calculation

- Taking account of creep

For loads with a duration causing creep, the total deformation including creep may be calculated by using an effective modulus of elasticity for concrete:

$$E_{c,eff} = E_{cm} / [1 + \varphi(\infty, t_0)] \quad (7.20)$$

where:

$\varphi(\infty, t_0)$ is the creep coefficient

Checking deflections by calculation

- Most rigorous method of assessing deflections:

Compute the **curvatures** at frequent sections along the member

& calculate the deflection by numerical integration

Do this twice,

1st time: assuming the whole member to be uncracked (Condition I)

2nd time: assuming the member to be fully cracked (Condition II)

then **interpolate** using $\alpha = \zeta \alpha_{II} + (1 - \zeta) \alpha_I$

Section 7 Serviceability limit states (SLS)

Deflection control

Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation

Using $\alpha = \zeta \alpha_{II} + (1 - \zeta) \alpha_I$ with deformation parameter $\alpha =$ reverse stiffness $1/EI$

Per finite mesh element, an equivalent stiffness $(EI)_r$ is calculated:

$$(EI)_r = \frac{1}{\frac{\zeta}{(EI)_{II}} + \frac{1-\zeta}{(EI)_I}} \quad (\zeta = 0 \text{ for uncracked sections})$$

$(EI)_I$: short term stiffness (uncracked condition)

$E = E_{cm}$ $I =$ based on total concrete css + reinf. area

$(EI)_{II}$: long term stiffness (fully cracked condition)

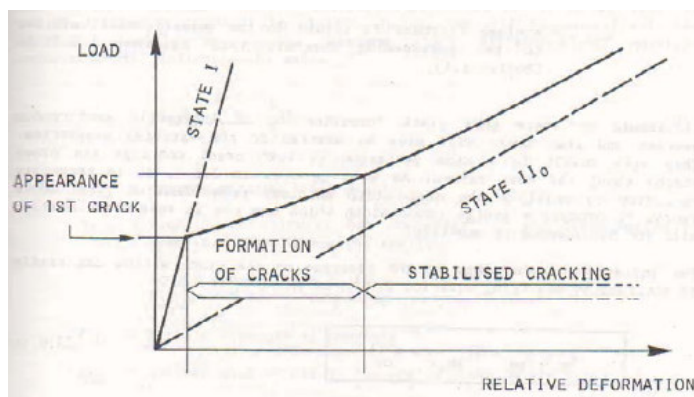
$E = E_{c,eff}$ $I =$ based on concrete css under compression + reinf. area

Section 7 Serviceability limit states (SLS)

Deflection control

Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation

The transition from the uncracked state (I) to the cracked state (II) does not occur abruptly, but gradually. From the appearance of the first crack, realistically, a parabolic curve can be followed which approaches the line for the cracked state (II).



→ distribution coefficient ζ

$$(EI)_r = \frac{1}{\frac{\zeta}{(EI)_{II}} + \frac{1-\zeta}{(EI)_I}}$$

Section 7 Serviceability limit states (SLS)

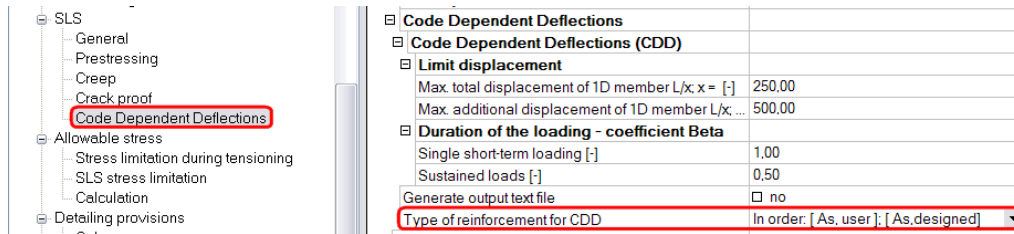
Deflection control



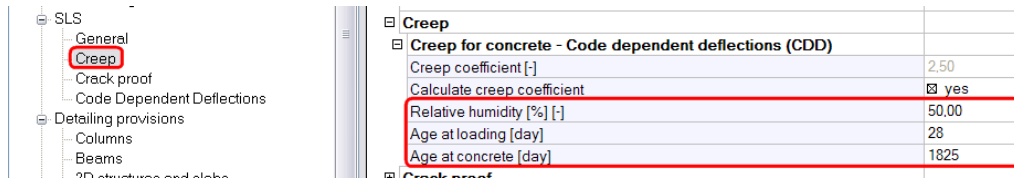
Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation

User input in Scia Engineer

- The type of reinforcement for which the CDD calculation will be performed



- The parameters for calculation of the creep coefficient (acc. to EN Annex B1)

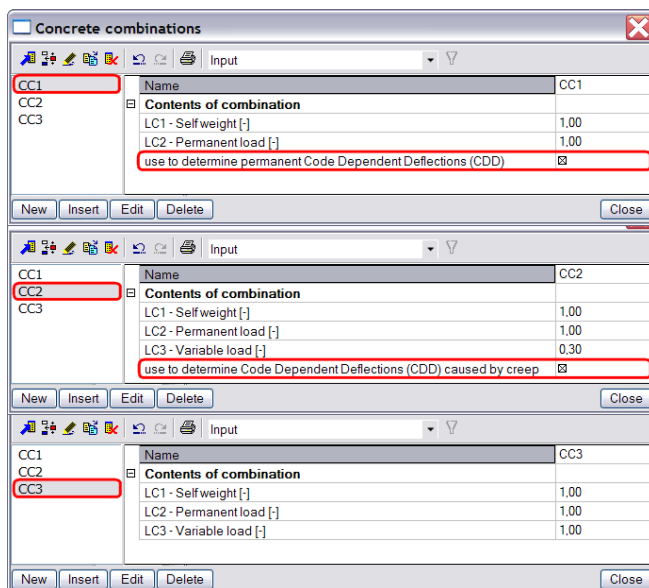


Section 7 Serviceability limit states (SLS)

Deflection control



Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation



Concrete combinations

CC1 (Immediate effect)

1,00 SW + 1,00 PL

CC2 (Creep effect)

1,00 SW + 1,00 PL + 0,30 VL

CC3 (Total effect)

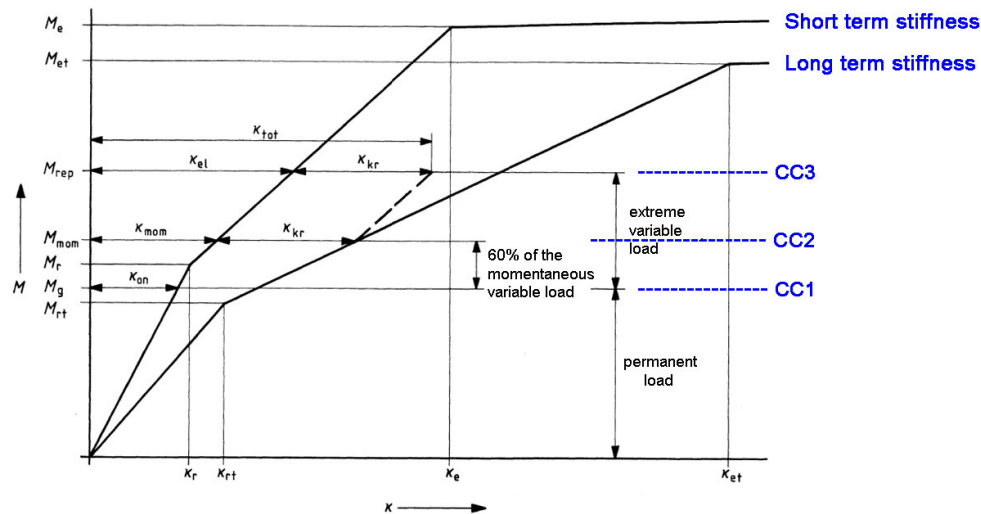
1,00 SW + 1,00 PL + 1,00 VL

Section 7 Serviceability limit states (SLS)

Deflection control

Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation

3 Concrete combinations ~ M_k diagram used by NEN (Dutch code)



Section 7 Serviceability limit states (SLS)

Deflection control

CDD versus PNL calculation in Scia Engineer

- CDD (Code Dependent Deflection) calculation
 - The formulas take into account the influence of cracks and creep
 - Quasi non-linear calculation: EI is calculated according to approximate formulas
 - Code dependent
- PNL (Physical Non Linear) calculation
 - Takes into account the non-linear behaviour of materials, the influence of cracks and creep
 - Real non-linear calculation: EI is calculated iteratively
 - Code independent

Section 8 Detailing of reinforcement - General

Section 8 Detailing of reinforcement - General

Spacing of bars

Min. bar spacing s_{\min}

The minimum clear distance (horizontal and vertical) between parallel bars should be

$$s_{\min} = \max \{ k_1 \cdot \phi ; (d_g + k_2 \text{ mm}) ; 20 \text{ mm} \}$$

where:

d_g is the maximum aggregate size

$k_1 = 1 \text{ mm}$ (recommended value)

$k_2 = 5 \text{ mm}$ (recommended value)

... such that the concrete can be placed and compacted satisfactorily (by vibrators) for the development of adequate bond

Section 8 Detailing of reinforcement - General

Permissible mandrel diameters for bent bars

Min. mandrel diameter $\phi_{m,min}$

$\phi_{m,min} =$ (recommended values, EN Table 8.1N)

a) for bars and wire

Bar diameter	Minimum mandrel diameter for bends, hooks and loops (see Figure 8.1)
$\phi \leq 16$ mm	4ϕ
$\phi > 16$ mm	7ϕ

b) for welded bent reinforcement and mesh bent after welding

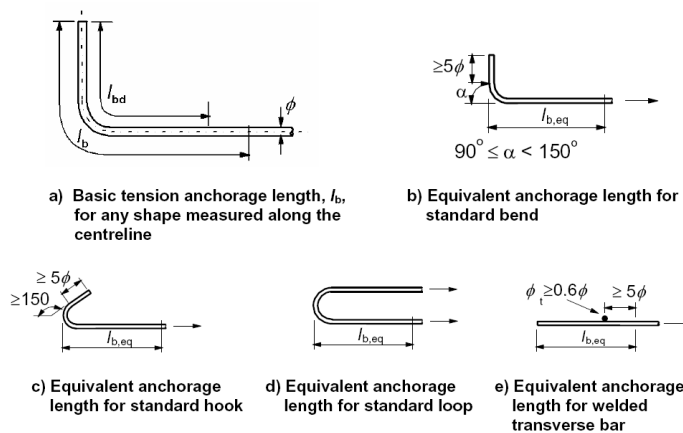
Minimum mandrel diameter	
5ϕ	$d \geq 3\phi$: 5ϕ $d < 3\phi$ or welding within the curved zone: 20ϕ
Note: The mandrel size for welding within the curved zone may be reduced to 5ϕ where the welding is carried out in accordance with prEN ISO 17660 Annex B	

... to avoid bending cracks in the bar, and to avoid failure of the concrete inside the bend of the bar

Section 8 Detailing of reinforcement - General

Anchorage of longitudinal reinforcement

Methods of anchorage



... to ensure that the bond forces are safely transmitted to the concrete, avoiding longitudinal cracking or spalling

Section 8 Detailing of reinforcement - General

Anchorage of longitudinal reinforcement

Ultimate bond stress

Design value of ultimate bond stress f_{bd}

$$f_{bd} = 2,25 \eta_1 \eta_2 f_{ctd} \quad (8.2)$$

where:

f_{ctd} is the design value of concrete tensile strength

η_1 is a coefficient related to the quality of the bond condition and the position of the bar during concreting:

$$\eta_1 = 1,0 \quad (\text{'good' conditions})$$

$$\eta_1 = 0,7 \quad (\text{all other cases})$$

η_2 is related to the bar diameter:

$$\eta_2 = 1,0 \quad \text{for } \phi \leq 32 \text{ mm}$$

$$\eta_2 = (132 - \phi)/100 \quad \text{for } \phi > 32 \text{ mm}$$

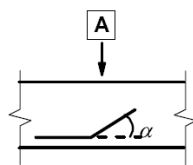
The ultimate bond strength shall be sufficient to prevent bond failure.

Section 8 Detailing of reinforcement - General

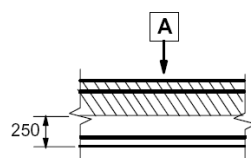
Anchorage of longitudinal reinforcement

Ultimate bond stress

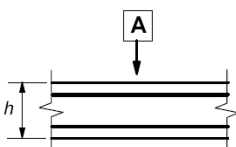
Description of bond conditions



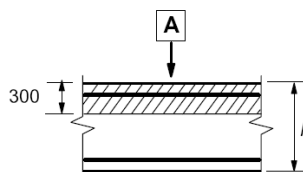
a) $45^\circ \leq \alpha \leq 90^\circ$



c) $h > 250 \text{ mm}$ A Direction of concreting



b) $h \leq 250 \text{ mm}$



d) $h > 600 \text{ mm}$

a) & b) 'good' bond conditions for all bars

c) & d) unhatched zone – 'good' bond conditions
hatched zone – 'poor' bond conditions

Basic anchorage length

Basic required anchorage length $l_{b,rqd}$

$$l_{b,rqd} = (\phi / 4) \cdot (\sigma_{sd} / f_{bd}) \quad (8.3)$$

... for anchoring the force $A_s \cdot \sigma_{sd}$ in a straight bar, assuming constant bond stress f_{bd} and where σ_{sd} is the design stress of the bar at the position from where the anchorage is measured from

Design anchorage length

Min. anchorage length $l_{b,min}$

$$l_{b,min} \geq \max \{ 0,3 l_{b,rqd} ; 10\phi ; 100 \text{ mm} \} \quad (\text{anchorages in tension}) \quad (8.6)$$

$$l_{b,min} \geq \max \{ 0,6 l_{b,rqd} ; 10\phi ; 100 \text{ mm} \} \quad (\text{anchorages in compression}) \quad (8.7)$$

Design anchorage length l_{bd}

$$l_{bd} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 l_{b,rqd} \geq l_{b,min} \quad (8.4)$$

where

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are coefficients given in EN Table 8.2,

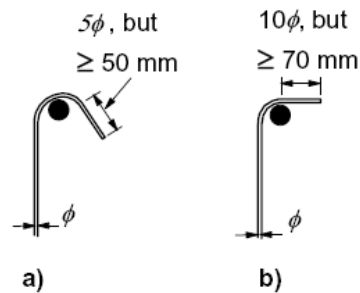
depending on shape of bar, concrete cover, type of confinement

Section 8 Detailing of reinforcement - General

Anchorage of links and shear reinforcement

Methods of anchorage

- by means of bends and hooks, or by welded transverse reinforcement
- a bar should be provided inside each hook or bend

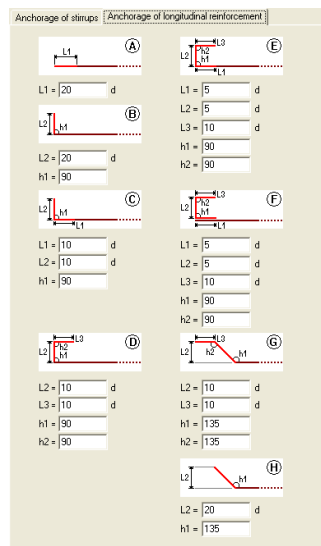


Section 8 Detailing of reinforcement - General

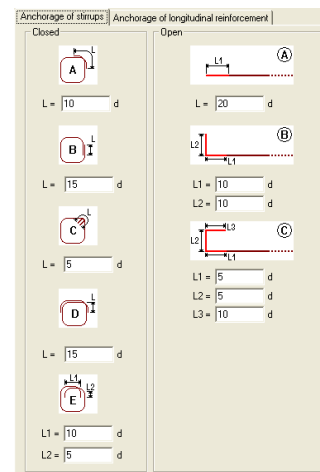
Anchorage of reinforcement

Anchorage types in Scia Engineer

- Longitudinal reinforcement



Stirrup reinforcement



Specifications

- unless stated otherwise, same rules as for individual bars apply
- all the bars in a bundle should have the same characteristics (type and grade) & similar sizes: max. ratio of diameters = 1,7
- in design, the bundle is replaced by a **notional bar** having the same sectional area and the same centre of gravity as the bundle + an equivalent diameter

Equivalent diameter ϕ_n

$$\phi_n = \phi \sqrt{n_b} \leq 55 \text{ mm} \quad (8.14)$$

where

n_b is the number of bars in the bundle,

$n_b \leq 4$ (for vertical bars in compression & bars in a lapped joint)

$n_b \leq 3$ (for all other cases)

Section 9 Detailing of members & particular rules

Longitudinal reinforcement

Min. reinforcement area $A_{s,min}$

$$A_{s,min} = 0,26 (f_{ctm}/f_{yk}) b_t d \geq 0,0013 b_t d \quad (\text{recommended value, 9.1N})$$

where:

b_t is the mean width of the tension zone

f_{ctm} according to EN Table 3.1

See also EN Section 7 for $A_{s,min}$ to control cracking.

Max. reinforcement area $A_{s,max}$ (outside lap locations)

$$A_{s,max} = 0,04 A_c \quad (\text{recommended value})$$

→ Min. areas in order to prevent a brittle failure in the reinforcement steel;

Max. areas to prevent sudden failure of the concrete compression zone

Shear reinforcement

Stirrup angle α

α = between 45° and 90° to the longitudinal axis of the structural element

Min. shear reinforcement ratio

$$\rho_w = A_{sw} / (s b_w \sin\alpha) \geq \rho_{w,min} \quad (9.4)$$

where:

A_{sw} is the area of shear reinforcement within length s

s is the spacing of the shear reinforcement along the longitudinal axis of the member

b_w is the breadth of the web of the member

$$\rho_{w,min} = (0,08 \sqrt{f_{ck}}) / f_{yk} \quad (\text{recommended value, 9.5N})$$

Shear reinforcement

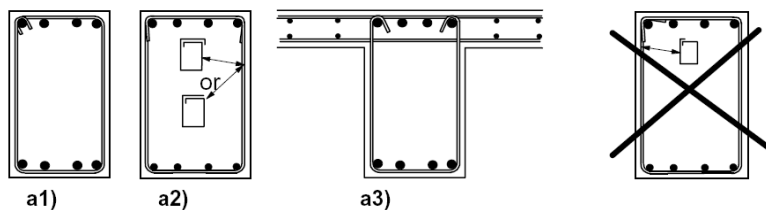
Max. longitudinal spacing $s_{l,max}$

$$s_{l,max} = 0,75 d (1 + \cot \alpha) \quad (\text{recommended value, 9.6N})$$

Max. transverse spacing of the legs $s_{t,max}$

$$s_{t,max} = 0,75 d \leq 600 \text{ mm} \quad (\text{recommended value, 9.8N})$$

Torsion reinforcement



a) recommended shapes

b) not recommended shape

Longitudinal spacing of the **torsion links** should not exceed

$$u / 8$$

$$s_{l,max} = 0,75 d (1 + \cot \alpha) \quad (\text{recommended value, 9.6N})$$

the lesser dimension of the beam cross-section

The **longitudinal bars** have to be distributed uniformly, with max. spacing of 350 mm.

Flexural reinforcement

Min. & max. reinforcement areas $A_{s,min}$ & $A_{s,max}$

Same requirements as for beams (see EN § 9.2)

Max. spacing $s_{max,slabs}$

$s_{max,slabs} =$ (recommended values)

- for the principal reinforcement: $3h \leq 400$ mm
 - for the secondary reinforcement: $3,5h \leq 450$ mm
- where h is the total depth of the slab

In areas with concentrated loads or areas of maximum moment:

- for the principal reinforcement: $2h \leq 250$ mm
- for the secondary reinforcement: $3h \leq 400$ mm

Flexural reinforcement

Curtailment of longitudinal tension reinforcement

Same requirements as for beams (EN § 9.2):

- calculate additional tensile force, ΔF_{td} , or
- estimate ΔF_{td} by shifting the moment curve a distance $a_1 = d$

One way slabs

Minimum secondary transverse reinforcement = 20% of the principal reinforcement

Shear reinforcement

Min. slab thickness

Minimum depth for a slab in which shear reinforcement is provided = 200 mm

Min. shear reinforcement ratio

Same requirements as for beams (see EN § 9.2)

Max. longitudinal spacing

$$s_{l,max} = 0,75 d (1 + \cot \alpha) \quad (9.9)$$

Max. transverse spacing

$$s_{t,max} = 1,5 d$$

Punching shear reinforcement

Where punching shear reinforcement is required:

Min. area of a link leg (or equivalent) $A_{sw,min}$

$$A_{sw,min} = [0,08 \sqrt{f_{ck}} \cdot (s_r \cdot s_t)] / [f_{yk} \cdot (1,5 \sin \alpha + \cos \alpha)] \quad (9.11)$$

where :

α is the angle between the shear reinforcement and the main steel

s_r is the spacing of shear links in the radial direction

s_t is the spacing of shear links in the tangential direction

Min. number of perimeters of link legs

= 2

Punching shear reinforcement

Distance between the face of the support and the first link leg perimeter

$$\geq 0,3 d \quad \text{and} \quad \leq 0,5 d$$

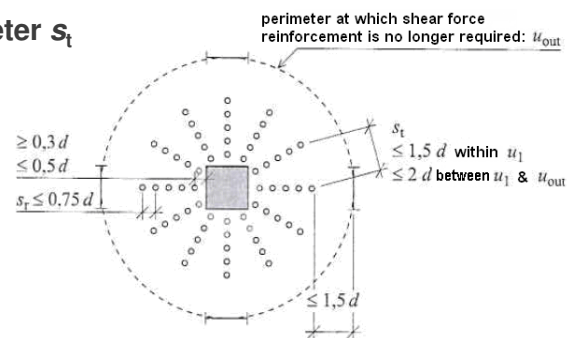
Max. radial spacing of the link leg perimeters s_r

$$s_r \leq 0,75 d$$

Max. tangential spacing around a perimeter s_t

$$s_t \leq 1,5 d \quad (\text{within } u_1)$$

$$s_t \leq 2 d \quad (\text{between } u_1 \text{ \& } u_{out})$$



Longitudinal reinforcement

Min. number of bars

At least one bar at each corner / Min. 4 bars for circular cross-sections

Min. diameter $\phi_{l,min}$

$$\phi_{l,min} = 8 \text{ mm} \quad (\text{recommended value})$$

Min. reinforcement area $A_{s,min}$

$$A_{s,min} = \max \{ 0,10 N_{Ed} / f_{yd} ; 0,002 A_c \} \quad (\text{recommended value, } 9.12N)$$

where:

N_{Ed} is the design axial compression force

Max. reinforcement area $A_{s,max}$

$$A_{s,max} = 0,04 A_c \quad (\text{outside lap locations}) \quad (\text{recommended value})$$

$$A_{s,max} = 0,08 A_c \quad (\text{at lap locations})$$

Transverse reinforcement

Min. diameter $\phi_{t,min}$

$$\phi_{t,min} = \max \{ 6 \text{ mm} ; 0,25 \phi_{l,max} \text{ applied} \}$$

Max. spacing along the column $s_{cl,tmax}$

$$s_{cl,tmax} \quad (\text{outside lapped joints}) \quad (\text{recommended value})$$

$$= \min \{ 20 \phi_{l,min} \text{ applied} ; \text{the lesser column dimension} ; 400 \text{ mm} \}$$

$$s_{cl,tmax} \quad (\text{near lapped joints if } \phi_{l,max} \text{ applied} > 14\text{mm}$$

$$\quad \& \text{ in the vicinity of a beam or slab})$$

$$= 0,6 \cdot \min \{ 20 \phi_{l,min} \text{ applied} ; \text{the lesser column dimension} ; 400 \text{ mm} \}$$

- The reinforcement design for walls may be derived from a strut-and-tie model.
- For walls subjected predominantly to out-of-plane bending the rules for slabs apply.

Vertical reinforcement

Min. reinforcement area $A_{s,vmin}$

$$A_{s,vmin} = 0,002 A_c \quad (\text{recommended value})$$

Max. reinforcement area $A_{s,vmax}$

$$A_{s,vmax} = 0,04 A_c \quad (\text{outside lap locations}) \quad (\text{recommended value})$$

$$A_{s,vmax} = 0,08 A_c \quad (\text{at lap locations})$$

Max. bar spacing s_{vmax}

$$s_{vmax} = \min \{ 3 \cdot \text{wall thickness} ; 400 \text{ mm} \}$$

Horizontal reinforcement

To be provided at each surface

Min. reinforcement area $A_{s,hmin}$

$$A_{s,hmin} = \max \{ 0,25 A_{s,v \text{ applied}} ; 0,001 A_c \} \quad (\text{recommended value})$$

Max. bar spacing s_{hmax}

$$s_{hmax} = 400 \text{ mm}$$

Transverse reinforcement

If $A_{s,v \text{ applied}}$ (total area in the two faces) $> 0,02 A_c$

then transverse reinforcement should be provided in accordance with the requirements for columns (see EN § 9.5).

References

- Technical Committee CEN/TC250, *Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings*, (2004).
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- Braam, C.R., Lagendijk, P., *Constructieleer Gewapend Beton*, (2008), Cement&BetonCentrum 's-Hertogenbosch.
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